Computing charm and bottom quark masses in lattice QCD

ALPHA Collaboration

SFB/TR 9

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DESY, Zeuthen

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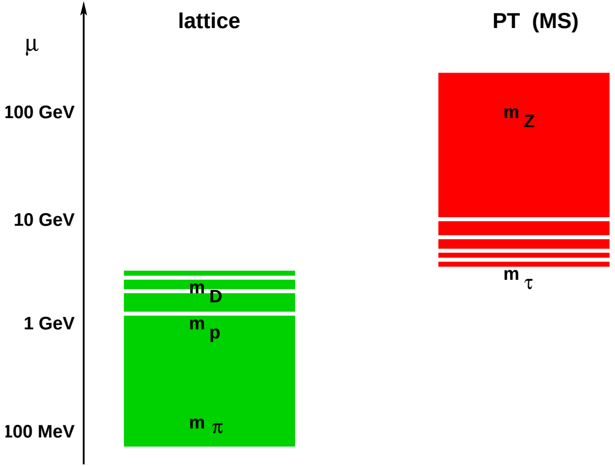
What do we want

$$\overbrace{\begin{bmatrix} F_\pi \\ m_\pi \\ m_\mathrm{K} \\ \mathbf{m_D} \\ \mathbf{m_B} \end{bmatrix}}^{\text{Experiment}} \quad \overset{\mathcal{L}_\mathrm{QCD}(g_0, m_f)}{\Longrightarrow} \quad \overbrace{\begin{bmatrix} \Lambda_\mathrm{QCD} \\ \hat{M} = (M_\mathrm{u} + M_\mathrm{d})/2 \\ M_\mathrm{s} \\ \mathbf{M_c} \\ \mathbf{M_b} \end{bmatrix}}^{\text{QCD parameters (RGI)}} \quad + \quad \overbrace{\begin{bmatrix} \xi \\ F_B \\ B_B \\ \dots \end{bmatrix}}^{\textbf{Predictions}}$$

Scale problem and strategy



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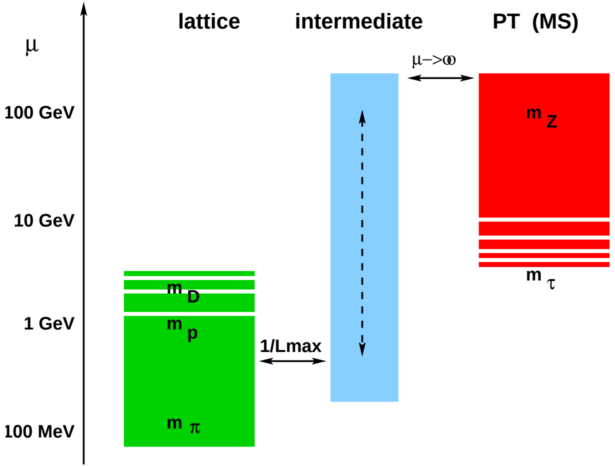


- intermediate: Schrödinger functional scheme

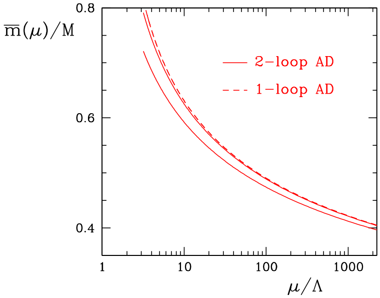
RGI quark masses

$$M = \lim_{\mu\to\infty} \overline{m}(\mu)[2b_0\bar{g}(\mu)^2]^{-d_0/2b_0}$$

$$\mu\frac{\partial\overline{m}}{\partial\mu} = \tau(\bar{g})$$

$$\tau(\bar{g}) \overset{\bar{g}\to 0}{\sim} -\bar{g}^2\{d_0+\bar{g}^2 d_1+\ldots\}\,,$$

$$d_0 = \frac{8}{(4\pi)^2}$$



- M scheme & scale independent:

$$\frac{\overline{m}_1(\mu)}{\overline{m}_2(\mu)} = 1+\mathrm{O}(\alpha(\mu))$$

$$\overset{\mu\to\infty}{\longrightarrow} 1 \quad\to\quad M_1 = M_2$$

Running mass, definition

- from the PCAC relation:

$$A_\mu^{sc} = \bar{s}\,\gamma_\mu\gamma_5\,c\,, \quad P^{sc} = \bar{s}\,\gamma_5\,c$$
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- precisely $\qquad g_0 \leftrightarrow a$

$$Z_\mathrm{A}(g_0)\,\langle D_s(p=0)|\,A_\mu^{sc}|0\rangle = (\overline{m}_\mathrm{c}(\mu) + \overline{m}_\mathrm{s}(\mu))\,Z_\mathrm{P}(\mu, g_0)\,\langle D_s(p=0)|\,P^{sc}|0\rangle$$

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- or

$$\underbrace{\overline{m}_\mathrm{c}(\mu)}_{\text{renormalized, running}} = \frac{Z_\mathrm{A}(g_0)}{Z_\mathrm{P}(\mu, g_0)}\;\underbrace{m_\mathrm{c}(g_0)}_{\text{bare, PCAC}}$$

scheme dependence from Z_P

The basic equation

$$\overline{m}_{\mathrm{c}}(\mu) = \frac{Z_{\mathrm{A}}(g_0)}{Z_{\mathrm{P}}(\mu, g_0)} \underbrace{m_{\mathrm{c}}(g_0)}_{\text{bare, PCAC}}$$

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- physics input: bare charm mass in the Lagrangian s.t. $m_D/F_\pi =$ experiment
- $Z_M(g_0)$ is known non-perturbatively

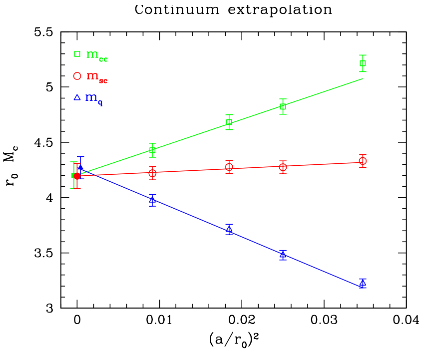
  for Wilson-type LQCD, $N_f = 0, 2$ ALPHA Collaboration SFB/TR 9

Charm quark mass

- remove $O(am_c)$ effects non-perturbatively [ALPHA Collaboration 2000]

  quenched [Sint & Rolf]

  charm just doable



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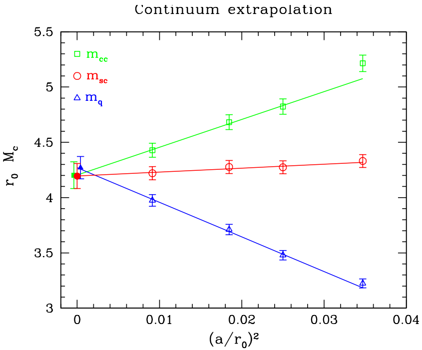

- $N_f = 0\,,\; r_0 = 0.5\,\mathrm{fm}$: $M_c = 1654(45)\mathrm{MeV} \rightarrow \overline{m}_c^{\overline{\mathrm{MS}}}(\overline{m}_c) = 1301(34)\mathrm{MeV}$

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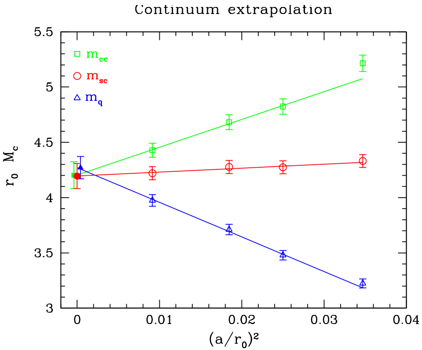

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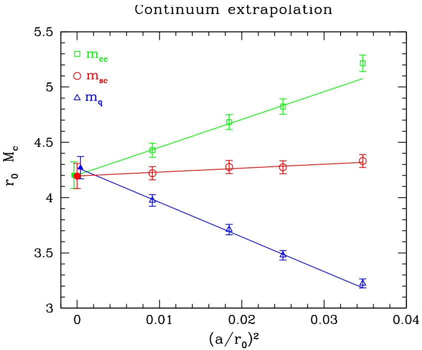

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- $aM_b \approx 4aM_c$!
  for b-quarks $a \rightarrow 0$ can't be controlled in this way $\rightarrow$ eff. theory

Effective theory for b-quark in heavy-light systems: HQET

[Caswell & Lepage ... Eichten & Hill ... ALPHA Collaboration]

$$\bar{b}(x)[\gamma_\mu D_\mu + m_\mathrm{b}]b(x) \quad \rightarrow$$

$$\mathcal{L}_\mathrm{HQET} = a^4 \sum_x \{\overline{\psi}_\mathrm{h}(x)[D_0 + \delta m]\psi_\mathrm{h}(x) + \underbrace{\omega_\mathrm{spin}}_{\sim 1/2m_\mathrm{b}} \overline{\psi}_\mathrm{h}(-\sigma\cdot\mathbf{B})\psi_\mathrm{h} + \underbrace{\omega_\mathrm{kin}}_{\sim 1/2m_\mathrm{b}} \overline{\psi}_\mathrm{h}(-\tfrac{1}{2}\mathbf{D}^2)\psi_\mathrm{h} + \ldots\}$$

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non-trivial matching problem

$$\omega_{\rm kin}\,,\ \omega_{\rm spin}\,,\ \delta m \quad \Leftrightarrow \quad M_{\rm b}$$

Mass renormalization in HQET

- static: $\overline{\psi}_\mathrm{h}(x)[D_0 + \delta m]\psi_\mathrm{h}(x)$

$$m_\mathrm{b}^{\overline{\mathrm{MS}}} = Z^{\overline{\mathrm{MS}}}\,[m_\mathrm{bare} + \delta m] \qquad m_\mathrm{bare} = \underbrace{m_\mathrm{B}}_{\mathrm{exp.}} - \underbrace{E_\mathrm{stat}}_{\text{“binding energy”}}$$

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- $1/m$ corrections make it worse: $a^{-1} \to a^{-2}$

Non-perturbative matching

- $\omega_{\rm kin}, \omega_{\rm spin}, \delta m + m_{\rm bare}$ from QCD [Heitger & S.]

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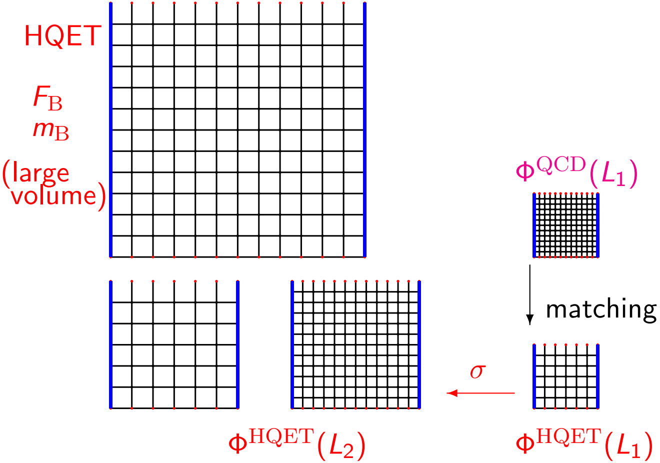
- $\Phi_i^{\rm QCD}(L) = \Phi_i^{\rm HQET}(L)\,, \quad i = 1, 2, 3$

The full strategy



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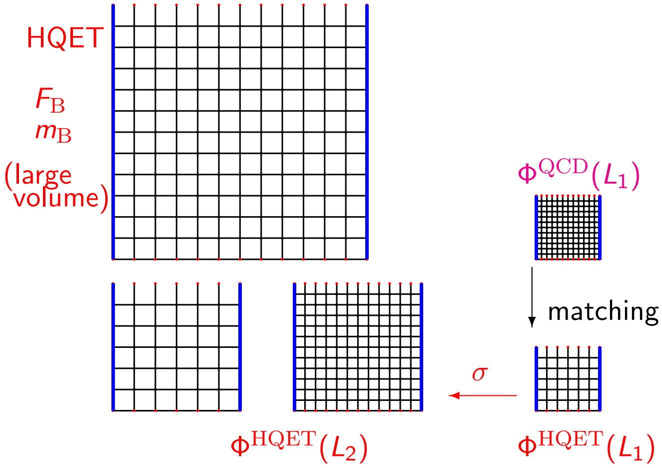

- continuum limit can be taken in all steps

Equation for $M_{\rm b}$ (static)

- pert: $m_{\rm B} = [Z^{\overline{\rm MS}}]^{-1}\, m_{\rm b}^{\overline{\rm MS}} + \underbrace{E_{\rm stat}}_{\text{“binding energy”}} + a^{-1}[1 + e_1 g_0^2 + \ldots]$

Equation for M_b (static)

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- NP:

$$m_\mathrm{B} = \underbrace{E_\mathrm{stat} - E_\mathrm{stat}^\mathrm{sub}}_{E_\mathrm{stat}-E_\mathrm{stat}(L_2)+E_\mathrm{stat}(L_2)-E_\mathrm{stat}(L_1)} + \underbrace{E_\mathrm{stat}^\mathrm{sub}}_{E(L_1,M_\mathrm{b})}$$

HQET QCD

$$L_1 \approx 0.4\,\mathrm{fm}\,, \quad L_2 = 2L_1$$

Example for continuum extrapolation (quenched)

$$m_{\rm B} = \underbrace{E_{\rm stat} - E_{\rm stat}(L_2) + E_{\rm stat}(L_2) - E_{\rm stat}(L_1)}_{a\to 0 \textbf{ in HQET}} + \underbrace{E(L_1, M_{\rm b})}_{\equiv \Phi_2/L_1}$$

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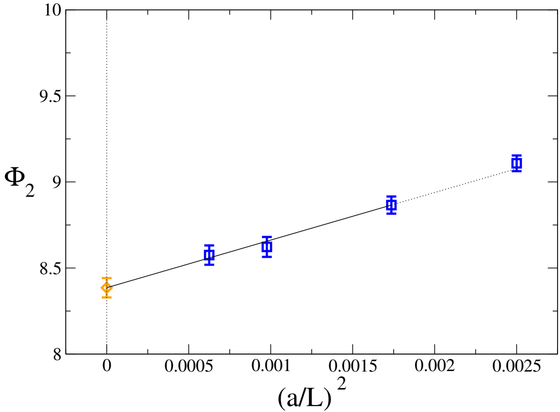
3 values of $M_{\rm b}$ example:

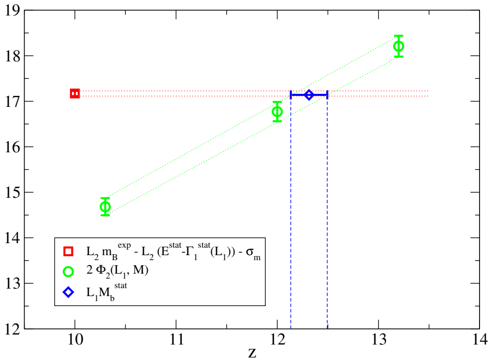
($M_{\rm b}$ from bare mass as before: $Z_{\rm M}$)



Result

in static approximation
express everything in units of $r_0 \approx 0.5\,\mathrm{fm}$ and solve:

$$\underbrace{r_0 m_\mathrm{B}}_{\text{experiment}} - r_0[E_\mathrm{stat} - E_\mathrm{stat}(L_2) + r_0[E_\mathrm{stat}(L_2) - E_\mathrm{stat}(L_1)] = \frac{r_0}{L_1}\Phi^\mathrm{QCD}(L_1, M_\mathrm{b})$$



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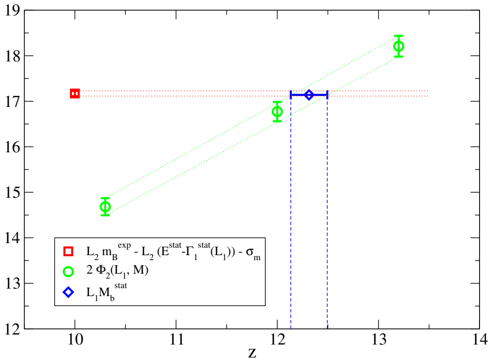

with $r_0 = 0.5\,\mathrm{fm}$

$$M_b^{\rm stat} = \begin{cases} 6771 \pm 99 \text{ MeV (HYP2)} \\ 6757 \pm 99 \text{ MeV (HYP1)} \end{cases}$$

and obtain the slope

$$S = \frac{1}{L_1}\frac{\partial\Phi^{\rm QCD}(L_1, M)}{\partial M} = 0.61(5)$$

error is dominated by the one on $Z_{\rm M}$.

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		$\theta_1 = 0$ $\theta_2 = 1/2$	$\theta_1 = 1/2$ $\theta_2 = 1$	$\theta_1 = 1$ $\theta_2 = 0$
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- related strategy, HQET only at lowest order, $1/m_{\rm b}$ corrections from QCD interpolations $\overline{m}_{\rm b}^{\overline{\rm MS}}(\overline{m}_{\rm b}) = 4.421(67)\,{\rm GeV}$ [Guazzini, S., Tantalo]

talk by Damiano Guazzini

Challenges, todos

- error for $M_{\rm b}$ dominated by $Z_{\rm M}(g_0)$ in relativistic QCD
  $\longrightarrow$ reduce by a factor 2 ?!
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- $M_{\rm c}$, $M_{\rm b}$ for $N_{\rm f} = 2$: in progress
- $N_{\rm f} = 3$ ...

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- no assumptions about (local) duality, "convergence" of the (divergent) pert. expansion, renormalon subtraction, ...