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RGI sum rule analysis for m_b

Adrian Signer

IPPP, Durham University

BASED ON WORK DONE IN COLLABORATION WITH A. PINEDA

overview

Introduction	• non-relativistic (large n) sum rules • compare large n and small n
Effective Theory	• outline of approach • renormalization-group improvement
Bottom Quark Mass Determination	• analysis • errors • resummation vs. higher order effects
Limit $n \to 0$	• consistency between large n and small n
Conclusions	

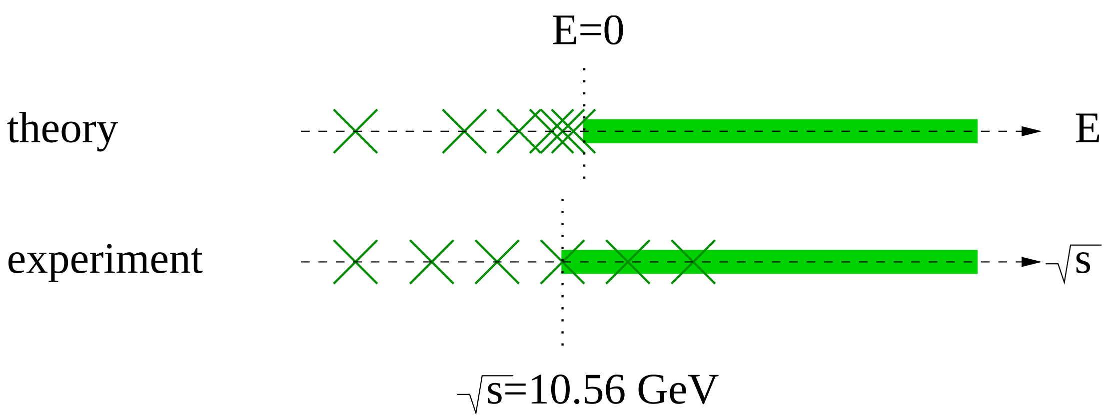
introduction

non-relativistic sum rules

$$M_n \equiv \frac{12\pi^2 e_b^2}{n!} \left(\frac{d}{ds}\right)^n \Pi(s)\Big|_{s=0} = \int \frac{ds}{s^{n+1}} R_{b\bar{b}}(s)$$

non-relativistic: n “large”, resonance contribution suppressed

define $s = q^2 = (E + 2m_b)^2$: n “large” $\leftrightarrow$ E “small”



introduction

non-relativistic system of bottom quark pair

problem with three scales:

- hard: m
- soft: $\vec{p} \sim mv \sim m\alpha_s$
- ultrasoft: $E = \sqrt{s} - 2m \sim mv^2 \sim m\alpha_s^2$

Hierarchy of scales:
$m \gg mv \gg mv^2 \gg \Lambda_{\rm QCD}$

mass of $\Upsilon(nS)$: $M_{\Upsilon(nS)} = 2m_b + E_n$ typical scale: $\mu \sim p \sim \alpha_s C_F m_b/n$

$\mu \sim 1.3\ \rm GeV$ for $n = 1$

dominant error non-perturbative

moments: $M_n = \int \frac{ds}{s^{n+1}} R_{b\bar{b}}(s)$ typical scale: $\mu \sim 2m_b/\sqrt{n}$

$\mu \sim 2.5\ \rm GeV$ for $n = 14$

dominant error perturbative

large n vs small n

- large n corresponds to small $v \sim 1/\sqrt{n}$, conventional fixed order (FO) perturbation theory breaks down (Coulomb singularity), use effective theory (ET)

R_{bb}	ET: LO	ET : NLO	ET : NNLO	...
FO: LO	$v\, c_{0,1}$	$v^2\, c_{0,2}$	$v^3\, c_{0,3}$	$v^4\, c_{0,4}$
FO: NLO	$\alpha\, c_{1,0}$	$\alpha\, v\, c_{1,1}$	$\alpha\, v^2\, c_{1,2}$	$\alpha\, v^3\, c_{1,3}$
FO: NNLO	$\alpha^2\, v^{-1}\, c_{2,-1}$	$\alpha^2\, c_{2,0}$	$\alpha^2\, v\, c_{2,1}$	$\alpha^2\, v^2\, c_{2,2}$
$\vdots$	$\alpha^3\, v^{-2}\, c_{3,-2}$	$\alpha^3\, v^{-1}\, c_{3,-1}$	$\alpha^3\, c_{3,0}$	$\alpha^3\, v\, c_{3,1}$

- cannot determine $\overline{m_b}(\mu)$ directly, have to use threshold mass
- perturbative series not very well behaved (large corrections)
- poorly determined continuum contribution of experimental moment strongly suppressed, only experimental input needed for experimental moments

introduction

determination of m_b with non-relativistic sum rules [Novikov et al.]

- done at NNLO [Hoang; Penin, Pivovarov; Melnikov, Yelkhovsky; Beneke, Signer]

$$R_{b\bar{b}} = v \sum_k \left(\frac{\alpha_s}{v}\right)^k \times \left\{1 \text{ (LO)}; \alpha_s, v \text{ (NLO)}; \alpha_s^2, v^2, \alpha_s v \text{ (NNLO)}\right\}$$

- problematic behaviour of perturbative series: large corretions, NNLO scale dependence larger than NLO scale dependence
- problems familiar from $t\bar{t}$ threshold $\to$ resummation of $\log v$

$$\sigma = v \sum_k \left(\frac{\alpha_s}{v}\right)^k \sum_l (\alpha_s \log v)^l \times \left\{1 \text{ (LL)}; \alpha_s, v \text{ (NLL)}; \alpha_s^2, v^2, \alpha_s v \text{ (NNLL)}\right\}$$

- since $v \sim \alpha_s$ we have $\log v \sim \log \alpha_s$, thus it is not surprising if resummation of $\log v$ has a large impact in bottom case as well.
  NOTE: in top case: $\log v \sim \log \mu_h/\mu_s \sim \log 175/30$

non-relativistic system of bottom quark pair

we have $v \sim 1/\sqrt{n} \ll 1$; count $\alpha_s \sim v \sim 1/\sqrt{n}$
thus need to resum $(\alpha/v)^k \sim (\alpha\sqrt{n})^k$ (Coulomb singularity)

non-relativistic expansion:

Coulomb Green function: $$\left(\frac{\Delta}{m^2} - C_F\frac{\alpha_s}{r} - E\right) G_c(\vec{r},\vec{r}\,',E) = \delta(\vec{r}-\vec{r}\,')$$

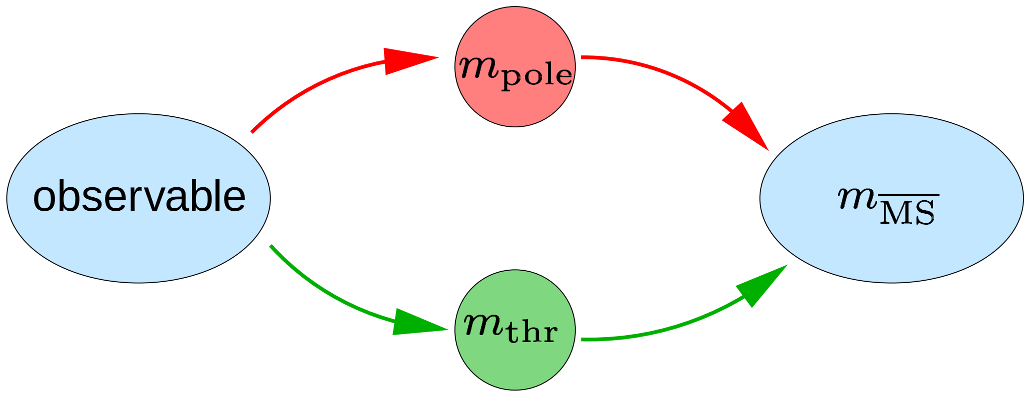
perturbation theory: $$G(\vec{r},\vec{r}\,',E) = G_c(\vec{r},\vec{r}\,',E) + \delta G(\vec{r},\vec{r}\,',E)$$

current: $$j^\mu \equiv \bar{Q}\gamma^\mu Q \to c_1\,\chi^\dagger\sigma^i\psi - \frac{c_2}{6m^2}\,\chi^\dagger\sigma^i\,(i\mathbf{D})^2\,\psi + \ldots$$

in pNRQCD : $$R(E) = \frac{24\pi e_q^2 N_c}{s}\left(c_1^2 - c_1 c_2\frac{E}{3\,m}\right)\,\mathrm{Im}\,G(0,0,E)$$

threshold mass

- pole mass is more IR sensitive (renormalon ambiguity) than other mass definitions $\to$ non-perturbative ambiguity $\sim \Lambda_{\rm QCD}$
- exploit that $V_{\rm Coul} + 2m_Q$ is not IR sensitive (renormalon cancellation) $\to$ threshold mass definitions [Bigi et.al; Beneke; Hoang et.al; Pineda]
- express observable in terms of threshold mass (here use PS mass [Beneke] and RS mass [Pineda])
- then relate threshold mass to $m_{\overline{\rm MS}}$; use four-loop relation (three-loop exact [Melnikov, Ritbergen; Chetyrkin, Steinhauser] and four-loop via large-β_0 approximation)



effective theory

- exploit $\alpha_s \ll 1$ and $v \sim 1/\sqrt{n} \ll 1$ $\rightarrow$ double expansion
- identify modes [Beneke, Smirnov] $\Rightarrow$ asymptotic expansion (method of regions)

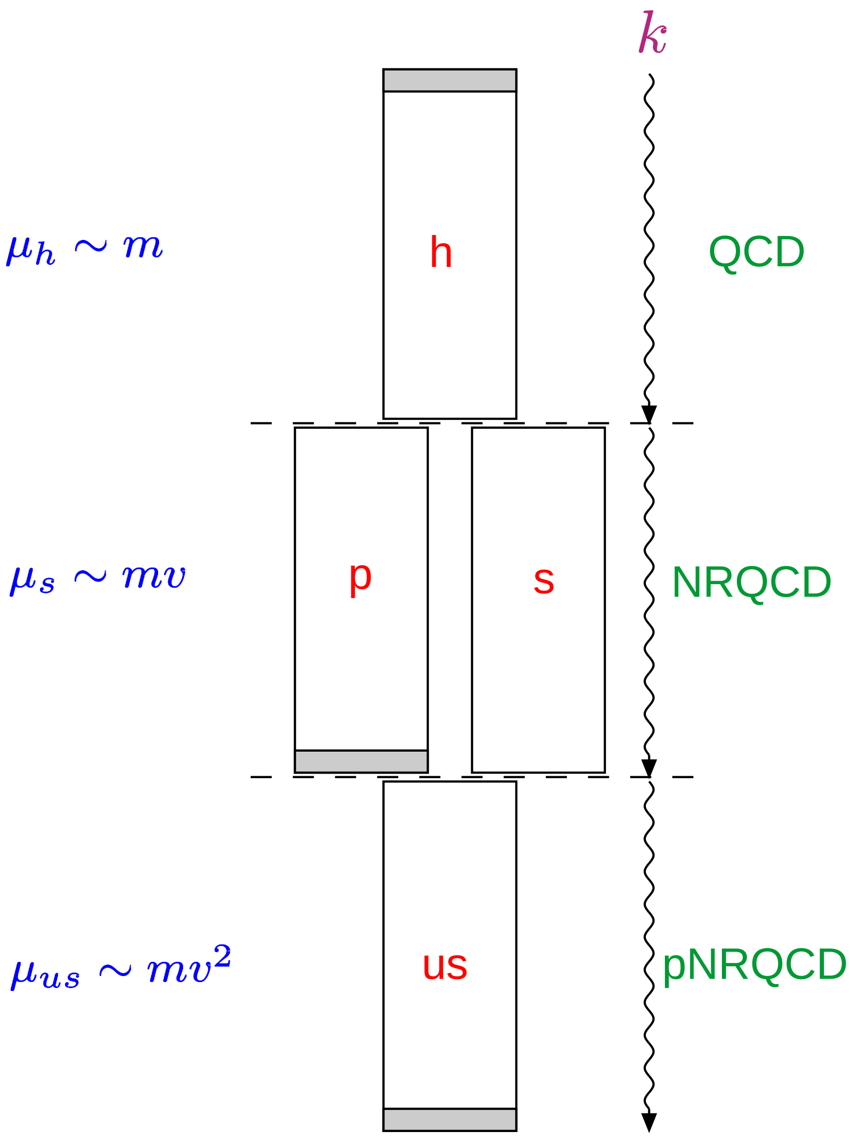
  hard $k^\mu \sim m$

  soft $k^\mu \sim mv$

  potential $k^0 \sim mv^2;\ \vec{k} \sim mv$

  ultrasoft $k^\mu \sim mv^2$
- integrate out 'unwanted' modes (final state described by potential quarks and ultrasoft gluons): [cp vNRQCD approach [Luke, Manohar, Rothstein]]
  QCD (h,s,p,u) $\longrightarrow$ NRQCD (s,p,u)$\longrightarrow$ pNRQCD (p$|_q$,u)
- done to NNLO [Beneke et.al; Hoang et.al; Melnikov et.al; Yakovlev; . . .] and NNLL [Hoang, Manohar, Teubner, Stewart; Pineda, Signer]
- matching of currents done at NLL and NNLO and partially at NNLL [Czarnecki, Melnikov; Beneke et.al; Pineda; Hoang et.al; Penin et.al]
- progress towards NNNLO [Brambilla et.al; Kniehl et.al; Beneke et.al]

effective theory



$\mathcal{L}_{\rm QCD}(\psi_h, \psi_s, \psi_p, g_h, g_s, g_p, g_{us})$

$\mathcal{L}_{\rm NRQCD}(\psi_s, \psi_p, g_s, g_p, g_{us})$
[Caswell, Bodwin, Braaten, Lepage]

resum $\log(\mu_h/\mu_s)$ via RGI matching coefficients [Bauer, Manohar, Pineda . . .]

$\mathcal{L}_{\rm pNRQCD}(\psi_p, g_{us})$
[Pineda, Soto]

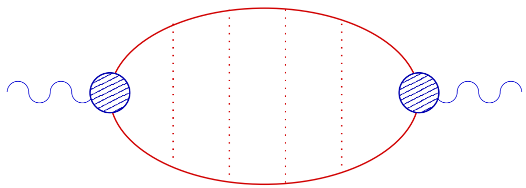
resum $\log \mu_s/\mu_{us}$ via RGI matching coefficients

effective theory

perturbation theory

We use dimensional regularization throughout, perform all calculations in momentum space and always use $\overline{\mathrm{MS}}$-subtraction [Beneke, AS, Smirnov]

$$G_c(\vec{r}, \vec{r}\,', E)\Big|_{\vec{r}=\vec{r}\,'=0} \equiv \int \frac{d^d\vec{p}}{(2\pi)^d}\frac{d^d\vec{p}\,'}{(2\pi)^d}\,\tilde{G}_c(\vec{p}, \vec{p}\,', E)$$



$$G_c(0,0,E) = -\frac{\alpha_s\, C_F\, m^2}{4\pi}\left(\frac{1}{2\lambda} + \frac{1}{2}\ln\frac{-4mE}{\mu^2} - \frac{1}{2} + \gamma_E + \psi(1-\lambda)\right)$$

where $\lambda \equiv C_F\,\alpha_s/(2\sqrt{-E/m})$; This sums all potential gluon (ladder) diagrams

for higher-order corrections evaluate single and double insertions

$$\delta G_c(0,0,E) = \int \prod \frac{d^d\vec{p}_i}{(2\pi)^d}\,\tilde{G}_c(\vec{p}_1, \vec{p}_2, E)\,\delta V(\vec{p}_2, \vec{p}_3)\,\tilde{G}_c(\vec{p}_3, \vec{p}_4, E)$$

bottom quark mass

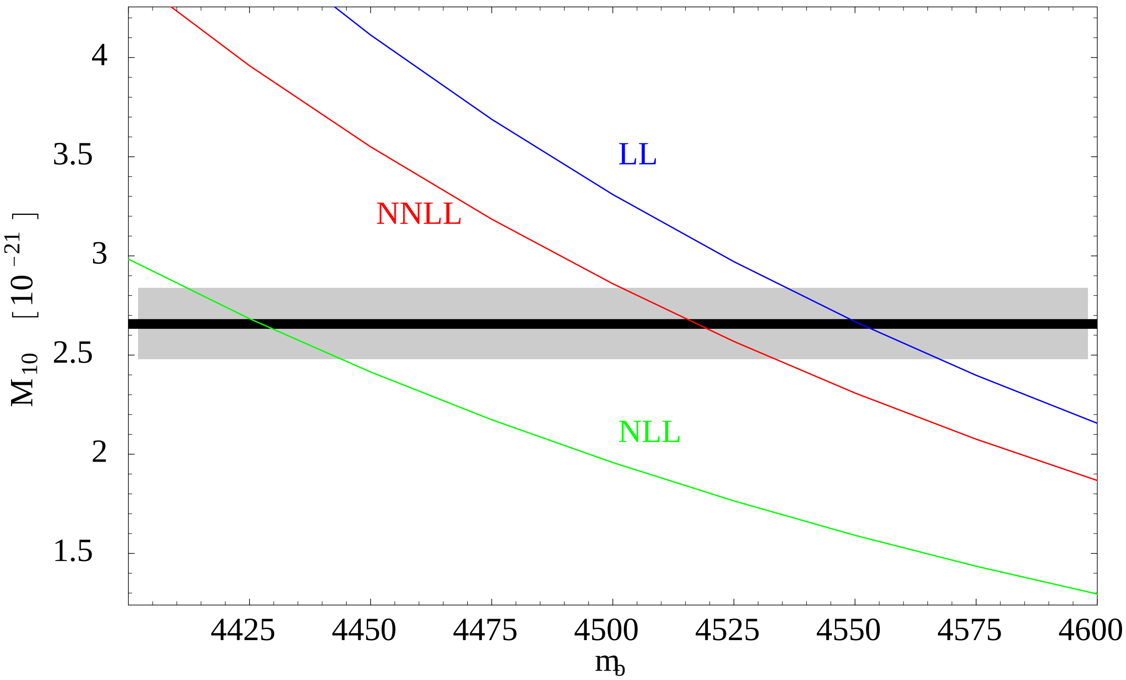
determination of m_b with non-relativistic sum rules

$$M_n = \int_{-\infty}^{\infty} \frac{2\,dE}{(E+2m_b)^{2n+1}} R_{bb}(E)$$
$$= \int_{-\infty}^{\infty} \frac{2\,dE}{(2m_b)^{2n+1}} e^{\frac{-nE}{m_b}} \left(1 - \frac{E}{2m_b} + \frac{nE^2}{4m_b^2} + \ldots\right) R_{bb}(E)$$

- determine theoretical moments via integration in complex plane
- typical scale $\mu_s \sim 2m_b/\sqrt{n}$, choose $n \leq 14$
- determine experimental resonanance moments of lowest six resonances (very well known) and continuum moments (poorly known), parametrize $R_{bb}^{cont} = 0.4 \pm 0.2$ [CLEO 1991]

bottom quark mass

RGI determination of m_b from M_{10} [Pineda, AS]



PS-mass

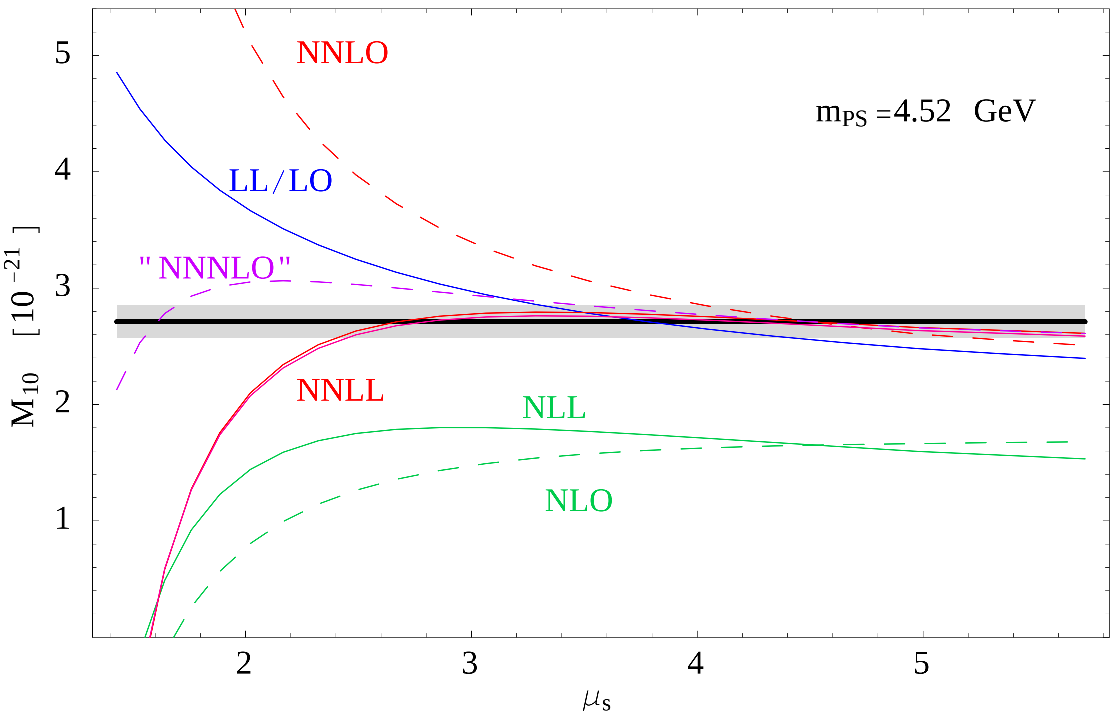
$\mu_F = 2$ GeV

$\mu = 2m_b/\sqrt{n}$

- corrections are large (though substantially smaller than without resummation)
- experimental error virtually irrelevant

bottom quark mass

RGI determination of m_b from M_{10} [Pineda, AS]



scale dependence

$\mu_F = 2$ GeV

$m_{PS} = 4.52$ GeV

- size of corrections reduced
- much improved μ_s scale dependence

} reduced theoretical error

errors in determination of threshold mass

experimental $\Delta_{\rm exp}$: mainly from $R_{b\bar{b}}^{\rm cont} = 0.4 \pm 0.2$ [CLEO, 1991]

coupling Δ_α : variation $0.115 < \alpha_s(M_Z) < 0.121$

theoretical $\Delta_{\rm th}$:

- do not use μ_s dependence (does not take into account size of corrections and μ_h and μ_{us} dependence)
- take half the size of the highest included correction, i.e. $\Delta_{\rm th} = |m_b^{(NNLL)} - m_b^{(NLL)}|/2$
- consistent with μ_h and μ_{us} scale dependence

total $\Delta_{\rm tot}$: add above partial errors in quadrature

"non-relativistic" $\Delta_{\rm nr}$:

- difference between expanded and non-expanded expression for moments
- this is not an independent error, only a consistency check

bottom quark mass

determination of $m_{b,\mathrm{PS}}(2\ \mathrm{GeV})$

use expanded version of moments

n	$m_{b,\mathrm{PS}}$(2 GeV)	Δ_{th}	Δ_{nr}	Δ_{exp}	Δ_{α}	Δ_{tot}	$\overline{m}_b$
6	4460	40	45	50	35	70	4135 ± 65
8	4505	45	15	25	30	60	4170 ± 55
10	4515	45	10	15	25	55	4185 ± 50
12	4520	45	10	10	20	50	4185 ± 45
14	4520	40	10	10	15	45	4185 ± 40

“combine”: single moment analysis

$$m_{b,\mathrm{PS}}(2\ \mathrm{GeV}) = 4.52 \pm 0.06\ \mathrm{GeV}$$

bottom quark mass

determination of $m_{b,\mathrm{RS}}(2\ \mathrm{GeV})$

use expanded version of moments

n	$m_{b,\mathrm{RS}}$(2 GeV)	Δ_{th}	Δ_{exp}	Δ_{α}	Δ_{tot}	$\overline{m}_b$
6	4315	55	50	25	80	4140 ± 70
8	4360	65	30	20	75	4180 ± 65
10	4370	65	30	10	70	4190 ± 60
12	4370	65	15	5	65	4190 ± 60
14	4370	65	10	5	65	4185 ± 55

“combine”: single moment analysis

$$m_{b,\mathrm{RS}}(2\ \mathrm{GeV}) = 4.37 \pm 0.07\ \mathrm{GeV}$$

bottom quark mass

determination of $\overline{\mathrm{MS}}$ mass $\overline{m}_b$

$$\overline{m_b} = 4.19 \pm 0.06\ \mathrm{GeV}$$

errors in conversion:

dependence on threshold mass : extract $\overline{m}_b$ also via $m_{b,\mathrm{PS/RS}}(1\ \mathrm{GeV})$
difference is $20/15\ \mathrm{MeV}$

error in conversion itself: take exact three-loop and estimated four-loop term
for error, drop last term $10/5\ \mathrm{MeV}$

total error: add above in quadrature

compare:

Melnikov, Yelkhovsky 1999:	$\overline{m_b} = 4.20 \pm 0.10\ \mathrm{GeV}$
Hoang 1999:	$\overline{m_b} = 4.20 \pm 0.06\ \mathrm{GeV}$
Beneke, Signer 1999:	$\overline{m_b} = 4.26 \pm 0.10\ \mathrm{GeV}$

limit $n \to 1$

when does non-relativistic approximation break down

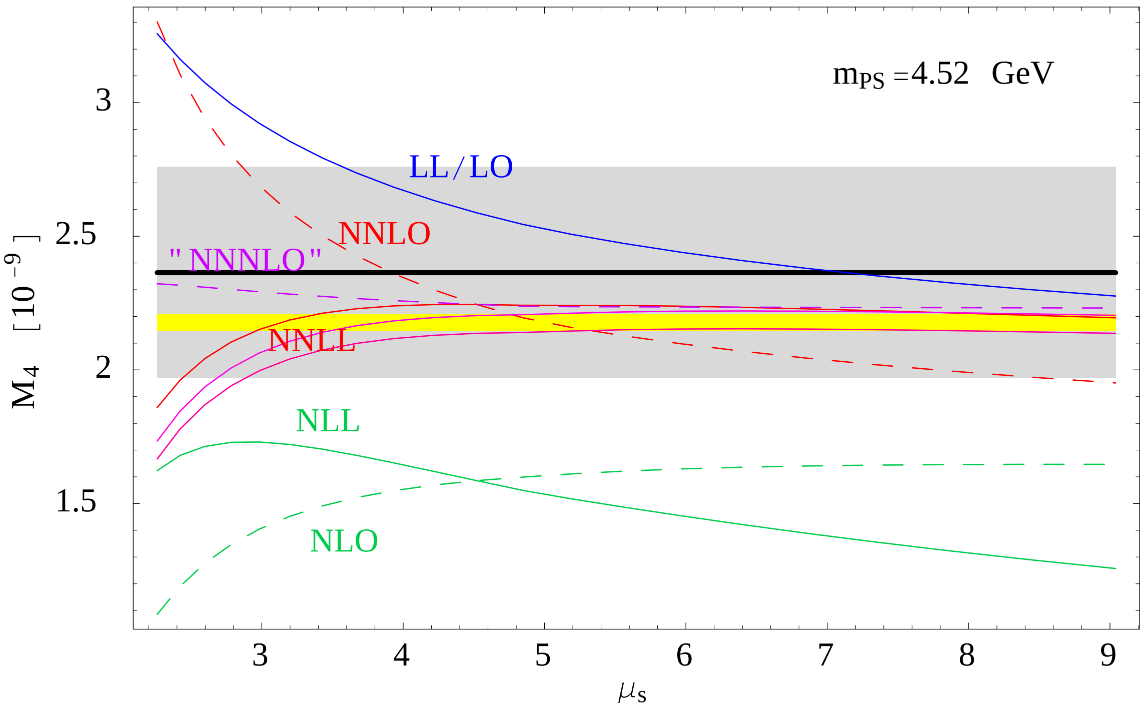
- for $n \leq 4$ use more refined determination of experimental moment [Kühn, Steinhauser, Sturm]
- do not use expanded version for theoretical moments

n	12	8	6	4	3	2
$m_{\rm PS}$ [GeV]	4.52	4.515	4.505	4.51	4.48	4.32

- consider different versions of non-relativistic expansion
- estimate importance of higher-order in v terms by adding corresponding terms of order α^2 in continuum part of R_{bb} [Chetyrkin et. al.]

limit $n \to 1$

determination of m_b from non-relativistic M_4 ?



scale dependence

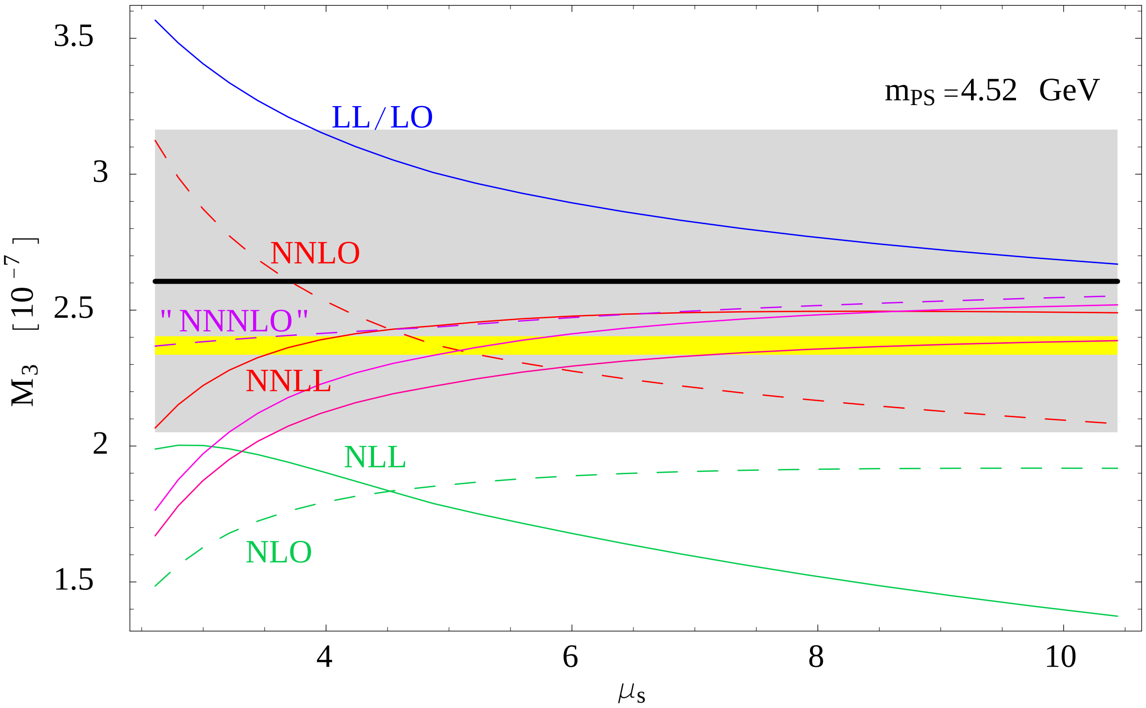
$\mu_F = 2$ GeV

$m_{\rm PS} = 4.52$ GeV

- importance of higher-order in v terms not dramatic $\delta m_b \sim 20$ MeV
- determination of experimental moment is critical

limit $n \to 1$

determination of m_b from non-relativistic M_3 ???

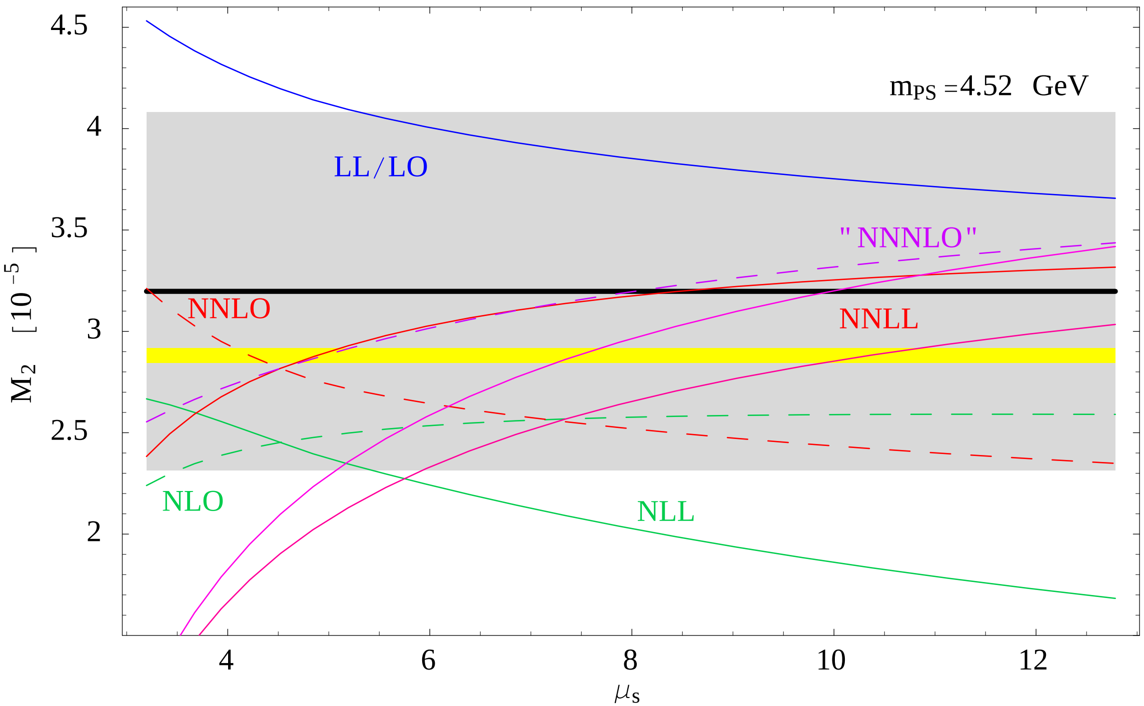


scale dependence

$\mu_F = 2$ GeV

$m_{PS} = 4.52$ GeV

- higher-order in v terms become important $\delta m_b \sim 60$ MeV determination of m_b from non-relativistic M_2 ???????



scale dependence

$\mu_F = 2 \text{ GeV}$

$m_{\text{PS}} = 4.52 \text{ GeV}$

- neglect of higher-order in v terms not acceptable

limit $n \to 1$

making the connection between large n and small n

For $n \lesssim 4$ full order $\alpha^0, \ \alpha^1$ and α^2 terms required, large-n approach breaks down

How important is the resummation of $(\alpha/v)^k$ for small moments?

Rough estimate:
consider effects of α^k terms in $G_c(0,0;E)$ with $k > 3 \ (4)$ in determination of m_b:

δm_b / n	8	4	3	2	1
$\delta m_b^{(3)}$ [MeV]	90	60	50	35	20
$\delta m_b^{(4)}$ [MeV]	40	20	10	7.5	5

For $n \gtrsim 4$ all order $(\alpha/v)^k$ terms required, fixed-order approach breaks down

ideally: combination of both

conclusions

summary:

- determination of bottom quark mass from non-relativistic sum rules yields $\overline{m_b} = 4.19 \pm 0.06\ \mathrm{GeV}$
- higher-order terms via resummation of $\log v$ have a large impact in the bottom case as well (not only in top) and result in an impressive improvement of the theoretical prediction
- large n sum rules are consistent with low n sum rules

further improvements:

- complete NNLL
- NNNLO.... resummation vs. higher-order ??
- combine standard fixed-order with effective theory results