Quark masses with automatically O(a) improved twisted fermions

Andrea Shindler
NIC/DESY Zeuthen

ETM Collaboration

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ETM Collaboration

- Germany — B. Blossier, F. Farchioni, K. Jansen, I. Montvay, K. Nagai, S. Schäfer, A. S., C. Tarantino, G. Münster, O. Bär
- Italy — T. Chiarappa, P. Dimopoulos, R. Frezzotti, G. Herdoiza, V. Lubicz, G. Martinelli, M. Papinutto, G.C. Rossi, L. Scorzato, S. Simula, A. Vladikas
- UK — C. McNeile, C. Michael, J. Pickavance, C. Urbach
- France — R. Baron, Ph. Boucaud, Z. Liu, B. Haas, O. Pène
- Spain — V. Gimenez, D. Palao
- Switzerland — U. Wenger
- Cyprus — D. Alexandrou, G. Koutsou

Outline

- twisted mass lattice QCD
  - tmQCD in the continuum
  - why Wtm on the lattice
- simulations with $N_f = 2$ degenerate light flavours
  - algorithmic improvements
  - setup and first results
- renormalization and quark masses

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- lattice is the only NP regularization of QCD
- test of QCD and phenomenology
- requires control on all the uncertainties

- Goal:
  - ⋆ low *up* and *down* quark masses ($M_\pi \lesssim 300$ MeV) Algorithm
  - ⋆ large volumes ($L > 2$ fm) Algorithm
  - ⋆ small discretization error ($O(a)$ improvement) Action
  - ⋆ NP renormalization Action

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twisted mass QCD (tmQCD)

- Twisted mass

$$S_F[\chi,\overline{\chi},G] = \int d^4x\, \overline{\chi} \left(\gamma_\mu D_\mu + m_q + i\mu_q\gamma_5\tau^3 \right) \chi,$$

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- mass term

$$m_q + i\mu_q\gamma_5\tau^3 = Me^{i\alpha\gamma_5\tau^3} \qquad M = \sqrt{m_q^2 + \mu_q^2}$$

$$\psi = \exp(i\omega\gamma_5\tau^3/2)\chi, \qquad \bar{\psi} = \bar{\chi}\exp(i\omega\gamma_5\tau^3/2),$$

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tmQCD ↔ QCD

Same theories in a different basis

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Wilson twisted mass

(Frezzotti, Grassi, Sint, Weisz: 2001)

$$S[\chi,\overline{\chi},U] = S_G[U] + S_F[\chi,\overline{\chi},U]$$

$$S_F[\chi,\overline{\chi},U] = a^4 \sum_x \overline{\chi}(x)\left[D_W + m_0 + i\mu_q\gamma_5\tau^3\right]\chi(x)$$

$$D_W = \frac{1}{2}\{\gamma_\mu(\nabla_\mu + \nabla^*_\mu) - ar\nabla^*_\mu\nabla_\mu\}$$

- angle between the Wilson and the mass term
- maximal disalignment between mass term and Wilson term
- chiral symmetry is recovered up to $O(a^2)$ only tuning $m_0 = m_c$

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$$\widetilde{G}(p) = \frac{-i\gamma_\mu\mathring{p}_\mu + \mathcal{M}(p) - i\mu_{\rm q}\gamma_5\tau^3}{\mathring{p}^2_\mu + \mathcal{M}(p)^2 + \mu^2_{\rm q}}$$

$$\mathring{p}_\mu = \frac{1}{a}\sin(ap_\mu) \quad \mathcal{M}(p) = m_0 + \frac{r}{2}a\hat{p}^2_\mu, \quad \hat{p}_\mu = \frac{2}{a}\sin(\frac{ap_\mu}{2})$$

$$\mathbf{p} = 0 \Rightarrow \mathring{p}^2_0 + m^2_0 + \frac{a^2r^2}{4}(\hat{p}^2_0)^2 + am_0r\hat{p}^2_0 + \mu^2_{\rm q}$$

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properties of Wtm

- infrared cutoff for any gauge background
- automatic O(a) improvement
- ease the renormalization of phenomenologically relevant operators
- O(a^2) flavour and parity breaking discretization errors

exceptional configurations cured

$$\gamma_5 D_{\rm W} \gamma_5 = D_{\rm W}^\dagger \qquad Q_W \equiv \gamma_5 (D_{\rm W} + m_0) = Q_W^\dagger$$

$$D = D_W + m_0 + i\mu_{\rm q}\gamma_5\tau^3 \qquad Q \equiv \gamma_5 D = Q_W + i\mu_{\rm q}\tau^3 \Rightarrow$$

$$Q^\dagger Q = Q_W^\dagger Q_W + \mu^2 = Q_W^2 + \mu^2$$

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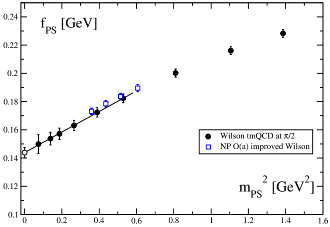

(Jansen, Papinutto, A.S, Urbach, Wetzorke: 2005)

Automatic O(a) improvement

Quark masses twisted mass fermions

Aachen - 22.02.07

A. Shindler

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- m.r. "physical" correlation functions are automatically O(a) improved if m_0 is tuned such that $m_{\rm R} = 0$
- Many proofs after the first one (Frezzotti, Martinelli, Papinutto, Rossi; A.S.; Sint; Aoki, Bär)
- "a good theoretical physicist should know how to solve a problem in 6-7 different ways" (R. P. Feynman)

$$\mathcal{O}_1 = i\bar\chi\sigma_{\mu\nu}F_{\mu\nu}\chi \qquad \mathcal{O}_2 = \mu_{\rm q}^2\bar\chi\chi \qquad \mathcal{O}_3 = \Lambda^2\bar\chi\chi$$

$$\langle\Phi\rangle = \langle\Phi\rangle_0 - a\int d^4y\langle\Phi\mathcal{L}_1(y)\rangle_0 + a\langle\Phi_1\rangle_0 + \ldots$$

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critical mass

(Aoki,Bär;Sharpe,Wu;Frezzotti et al.)

- from the proof apparently there is no bound on the value of $\mu_{\rm q}$
- to avoid big O(a^2) one needs to have $\mu_{\rm R} > a\Lambda^2$

$$\Lambda = 300\text{MeV} \quad a = 0.1\text{fm} \Rightarrow \mu_{\rm R} > 45\text{MeV}$$

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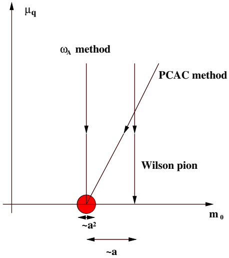

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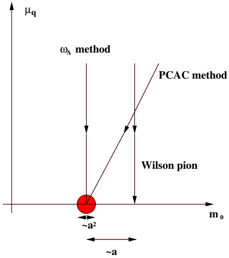

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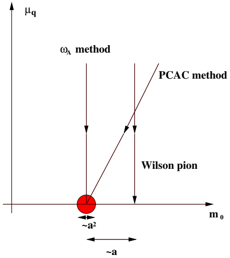

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Method	Definition
Wilson pion	$\lim_{m_0\to m_\mathrm{c}} M_\pi^2$
Wilson-clover pion	$\lim_{m_0\to m_\mathrm{c}} M_\pi^2$
Wilson-clover PCAC	$\lim_{m_0\to m_\mathrm{c}} m_\mathrm{PCAC}$
ω_A	$\langle A_\mu^a(x)P^a(y)\rangle = 0 \quad a=1,2$
ω_P	$\langle A_\mu^3(x)P^3(y)\rangle = 0$
PCAC	$\lim_{\mu_\mathrm{q}\to 0} m_\mathrm{c}(\mu_\mathrm{q})$

$$M_{\pi^\pm}^2 = 2B_0M'\left[1+\frac{2B_0M'}{32\pi^2f^2}\log(2B_0M'/\Lambda_\pi^2)\right] + 2aB_0M'\cos\omega'(2\delta_W-\delta_{\widetilde{W}})$$

$$+2a^2w'\cos^2\omega'$$

$$\mu_\mathrm{R} > a^2\Lambda^3 \Rightarrow \Lambda = 300\mathrm{MeV} \quad a=0.1\mathrm{fm} \Rightarrow \mu_\mathrm{R} > 7\mathrm{MeV}$$

- understand which are the symmetries that are recovered in the continuum if $m_\mathrm{R}=0$ and impose suitable identities on the lattice

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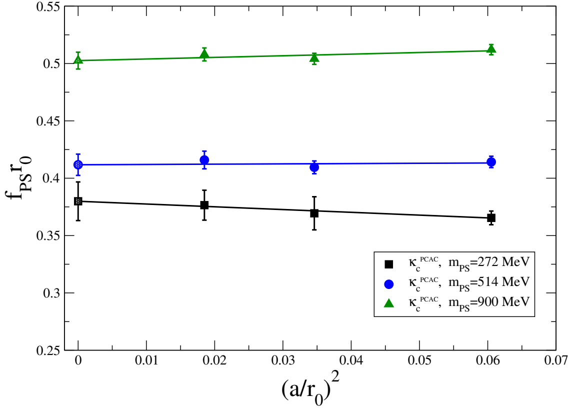
scaling test

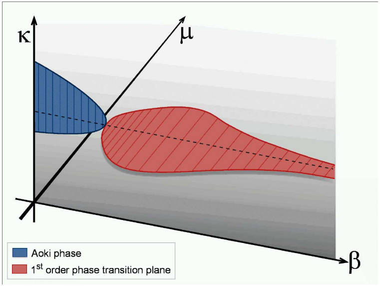
PCAC definition for the critical mass



(Jansen, Papinutto, A.S., Urbach, Wetzorke: 2005)

phase structure

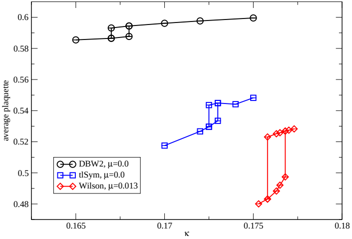
- Wilson-type fermions (plain and twisted) have a non-trivial phase structure at finite lattice spacing (Aoki; Sharpe, Singleton)
- The strength of the phase transition depends on details of the action
  - ⋆ gluonic: b_1
  - ⋆ fermionic: $c_{\rm sw}$



(Farchioni et al.: 2004-2005)

gauge action

$$S_g = \frac{\beta}{3} \sum_x \left[(1 - 8b_1) \sum_{\mu<\nu}^{4} \left(1 - \mathrm{ReTr} \left(U^{1\times 1}_{x,\mu,\nu} \right) \right) + b_1 \sum_{\mu\neq\nu}^{4} \left(1 - \mathrm{ReTr} \left(U^{1\times 2}_{x,\mu,\nu} \right) \right) \right]$$



(Farchioni et al.: 2004-2005)

- tree-level Symanzik improved (tlSym) gauge action ($b_1 = -1/12$)
- tlSym:
  - weakens the first order phase transitions compared to Wilson gauge action ($b_1 = 0$)
  - better scaling than DBW2 ($b_1 = -1.4088$)

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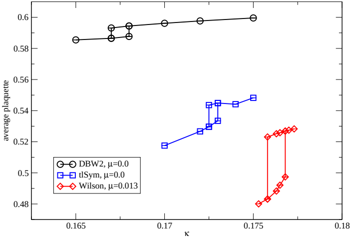

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- tlSym:
  - ⋆ weakens the first order phase transitions compared to Wilson gauge action ($b_1 = 0$)
  - ⋆ better scaling than DBW2 ($b_1 = -1.4088$)

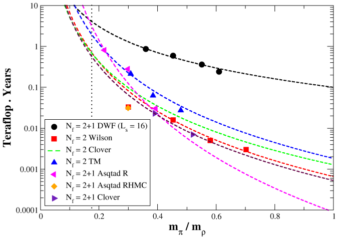
speeding up the HMC

- variant of the HMC algorithm

(C. Urbach, K. Jansen, A.S., U. Wenger: 2005)

  - ⋆ multiple time scale integration on top of a mass preconditioning (Hasenbusch, 2001)

- Other variants: all of them are efficient to reach small quark masses
  - ⋆ domain decomposition (Lüscher: 2003-2004)
  - ⋆ RHMC (Clark, Kennedy: 2006)
- Wilson fermions are back in the game



(Clark: 2006)

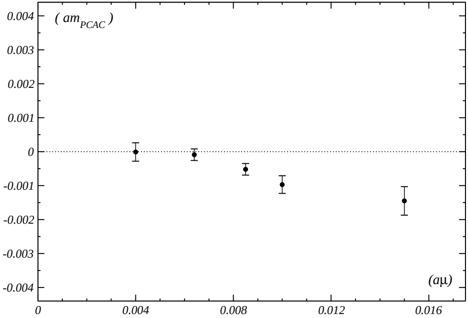
Simulations: plan

- fermion: $N_\mathrm{f} = 2$ Wtm at full twist
- gauge: tlSym
- three lattice spacings: $0.075 - 0.115$ fm
- $270 \lesssim m_\mathrm{PS} \lesssim 550$ MeV
- $L > 2$ fm

Simulations: setup

β	target a (fm)	$L^3 \cdot T$	$\kappa_{\rm crit}$	$a\mu$	$N_{\rm traj}$	target $m_{\rm PS}$ (MeV)
4.05	~ 0.075	$32^3 \cdot 64$	0.15701	0.0030	5000	~ 270
				0.0060	5000	~ 380
				0.0080	3000	~ 430
				0.0120	2000	~ 530
3.9	~ 0.09	$24^3 \cdot 48$	0.160856	0.0040	9400	~ 300
				0.0064	5000	~ 380
				0.0085	5000	~ 440
				0.0100	5000	~ 480
				0.0150	5000	~ 580
		$32^3 \cdot 64$		0.0040	5000	~ 300
3.8	~ 0.11	$20^3 \cdot 48$	0.164099	0.0060	3900	~ 290
				0.0090	3600	~ 390
				0.0120	5000	~ 450
				0.0150	3300	~ 510

full twist



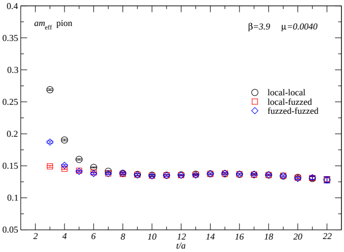
- $am_{\mathrm{PCAC}}(a\mu_{\mathrm{min}}) = -0.00001(27)$
- for all μ values: $am_{\mathrm{PCAC}} \lesssim (a\mu_{\mathrm{q}})(a\Lambda_{\mathrm{QCD}})$
- The weak μ_{q}-dependence of m_{PCAC} is an $\mathcal{O}(a)$ effect

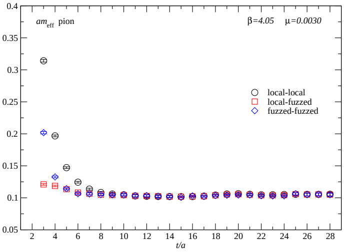
$\rightsquigarrow$ $\mathcal{O}(a^2)$ artifacts in physical quantities

Pion sector: correlators and effective masses

- quark propagator: stochastic sources to include all spatial sources
- Change the location of the time-slice source: reduce autocorrelations
- Fuzzing

$$C_{PP}(x_0) = a^3 \sum_{\mathbf{x}} \langle P^a(0) P^a(\mathbf{x}, x_0) \rangle \quad a = 1, 2$$





- stable masses $\rightsquigarrow$ isolate ground state from excited states
- small statistical errors

Ward identities

- the continuum WI are valid at finite lattice spacing up to discretization errors with renormalized operators

$$\partial_\mu A^a_\mu = 2m_{\rm q} P^a + i\mu_{\rm q}\delta^{3a}S^0$$
$$\partial_\mu V^a_\mu = -2\mu_{\rm q}\,\varepsilon^{3ab}P^b$$

- at finite lattice spacing there is a conserved vector current

$$\partial^*_\mu \langle \widetilde{V}^a_\mu(x)O(0)\rangle = -2\mu_{\rm q}\epsilon^{3ab}\langle P^b(x)O(0)\rangle \Rightarrow Z_\mu = \frac{1}{Z_P}$$

- it is possible to extract the PS decay constant without renormalization factors
- the resulting decay constant is automatically O(a) improved without any improvement coefficient

$$f_{\rm PS} = \frac{2\mu_{\rm q}|\langle\Omega|\hat{P}^a|{\rm PS}\rangle|}{M^2_{\rm PS}} \qquad a = 1, 2,$$

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Pion: decay constant and χPT fits

Can chiral perturbation theory (χPT) reproduce the data? $\beta = 3.9$

- we use continuum χPT to describe the dependence on:
  - ⋆ finite spatial size L
  - ⋆ the mass $\mu_{\rm q}$
- Simultaneous fit to $N_{\rm f} = 2$ χPT at NLO

(Gasser, Leutwyler, 1987; Colangelo *et al.*, 2005)

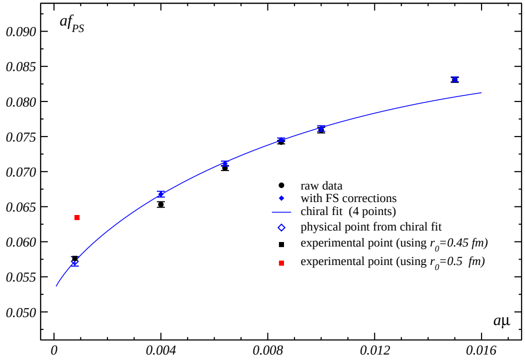
$$M_{\rm PS}^2(L) = 2B_0\mu_{\rm q}\left[1+\frac{1}{2}\xi\tilde{g}_1(\lambda)\right]^2\left[1+\xi\log(2B_0\mu_{\rm q}/\Lambda_3^2)\right],$$

$$f_{\rm PS}(L) = F\,[1-\xi\tilde{g}_1(\lambda)]\left[1-2\xi\log(2B_0\mu_{\rm q}/\Lambda_4^2)\right]$$

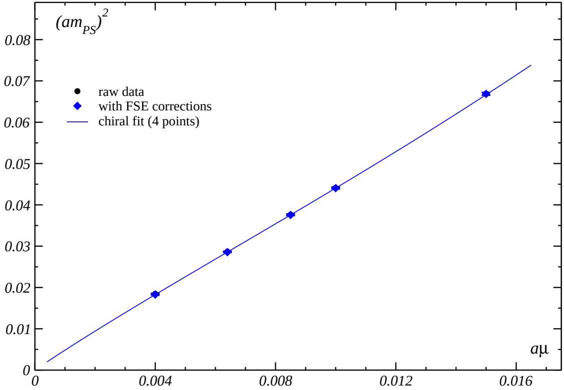
where $\xi = 2B_0\mu/(4\pi F)^2$, $\lambda = \sqrt{2B_0\mu L^2}$, $\tilde{g}_1(\lambda)$ is a known function

- fit parameters: B_0, F, Λ_3 and Λ_4
- extract low-energy constants: $\bar{l}_{3,4} \equiv \log(\Lambda_{3,4}^2/m_\pi^2)$

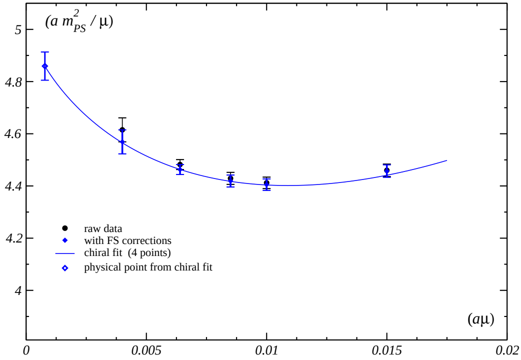
Pion sector: af_{PS} vs. $a\mu_q$ $\beta = 3.9$



Pion sector: (am_{PS}^2) vs. $a\mu$ $\beta = 3.9$



Pion sector: m_{PS}^2/μ vs. μ $\beta = 3.9$



Pion: results from χPT fits

$$M_{\rm PS}^2(L) = 2B_0\mu \left[1 + \frac{1}{2}\xi\tilde{g}_1(\lambda)\right]^2 \left[1 + \xi \log(2B_0\mu/\Lambda_3^2)\right]$$

$$f_{\rm PS}(L) = F\,[1 - \xi\tilde{g}_1(\lambda)] \left[1 - 2\xi \log(2B_0\mu/\Lambda_4^2)\right]$$

where $\xi = 2B_0\mu/(4\pi F)^2\,, \qquad \lambda = \sqrt{2B_0\mu L^2}$

$$2aB_0 = 4.99(6)$$

$$aF = 0.0534(6)$$

$$\log(a^2\Lambda_3^2) = -1.93(10)$$

$$\log(a^2\Lambda_4^2) = -1.06(4)$$

$$\chi^2/dof = 3.5/4 \sim 0.9$$

- The "physical point" $a\mu_\pi$ is determined by requiring $M_{\rm PS}/f_{\rm PS} = 139.6/130.7 = 1.068 \rightsquigarrow$ we get: $a\mu_\pi = 0.00078(2)$
- Taking $f_\pi = 130.7$ MeV, we obtain $a = 0.087(1)$ fm
- Using $r_0/a = 5.22(2)$ we get: $r_0 = 0.454(7)$ fm
- We determine : $\bar{l}_{3,4} \equiv \log(\Lambda_{3,4}^2/M_\pi^2)$

Pion: low-energy constants $\beta = 3.9$

Accurate determinations of $\bar{l}_{3,4} \equiv \log(\Lambda^2_{3,4}/M^2_\pi)$

$$\bar{l}_3 = 3.65 \pm 0.12$$

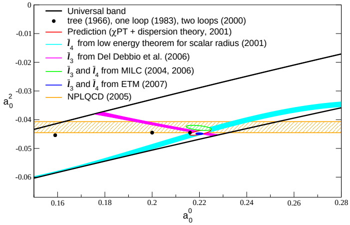
$$\bar{l}_4 = 4.52 \pm 0.06$$

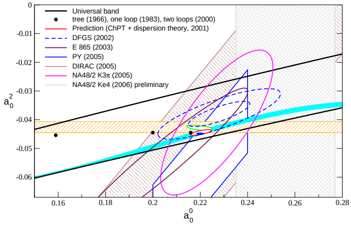
Other estimates (Leutwyler, hep-ph/0612112)

- $\bar{l}_3$:
  - ⋆ $\bar{l}_3 = 2.9 \pm 2.4$ from the mass spectrum of the pseudoscalar octet
  - ⋆ $\bar{l}_3 = 0.8 \pm 2.3$ from MILC
  - ⋆ $\bar{l}_3 = 3.0 \pm 0.6$ from lattice CERN group
- $\bar{l}_4$:
  - $\bar{l}_4 = 4.3 \pm 0.9$ from f_K/f_π
  - $\bar{l}_4 = 4.4 \pm 0.2$ from the radius of the scalar pion form factor
  - $\bar{l}_4 = 4.0 \pm 0.6$ from MILC

s-wave $\pi\pi$ scattering

(Leutwyler (priv. com.) :2007)





- scalar radius of the pion

$$\langle r^2 \rangle = 0.637(26)\mathrm{fm}^2 \quad [\mathrm{ETMC} : 2007]$$

$$\langle r^2 \rangle = 0.61(4)\mathrm{fm}^2 \quad [\mathrm{Colangelo, Gasser, Leutwyler} : 2001]$$

MOM scheme

(Martinelli, Pittori, Sachrajda, Testa, Vladikas : 1995)

- does not rely on bare perturbation theory
- first matching the lattice with an intermediate momentum subtraction (MOM) scheme – then passing to $\overline{MS}$ scheme
- the details of the MOM scheme are only of practical importance
- Landau gauge – renormalization conditions on propagators and vertex functions at some momentum p

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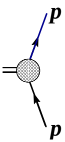
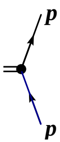


$$= Z_2^{\mathrm{MOM}} / Z_P^{\mathrm{MOM}} \times$$



$$Z_P(g_0, a\mu) = Z_P^{\mathrm{MOM}}(g_0, a\mu) X^{\mathrm{MOM}}(g_{\overline{MS}}, p/\mu)$$

MOM scheme

$$G_\Gamma^{ud}(p,p') = \sum_{x,y} \langle u(x)(\bar{u}\Gamma d)(0)\bar{d}(y)\rangle e^{-ipx+ip'y}$$

$$S_u(p) = \sum_x \langle u(x)\bar{u}(0)\rangle e^{-ipx}$$

$$\Gamma_\Gamma^{ud}(p,p') = \mathrm{Tr}\left[S_u(p)^{-1} G_\Gamma^{ud}(p,p') S_d(p')^{-1} \mathbb{P}_\Gamma\right]$$

$$Z_\Gamma Z_q^{-1} \Gamma_\Gamma^{u,d}(p,p)|_{p^2=\mu^2} = 1 \qquad Z_q \frac{i}{12} \mathrm{Tr}\left[\frac{p_\mu \gamma_\mu S(p)^{-1}}{p^2}\right]_{p^2=\mu^2} = 1$$

O(a) improvement

- off-shell quark Green function are not O(a) improved by the standard on-shell improvement program
- formulations of QCD with exact chiral symmetry have off-shell improvement

$$\gamma_5 D_{\mathrm{GW}}^{-1} + D_{\mathrm{GW}}^{-1}\gamma_5 = 2a\gamma_5$$

- with Wtm we retain a subgroup of the axial symmetry

$$\gamma_5\tau^1 D^{-1} + D^{-1}\tau^1\gamma_5 = 0$$

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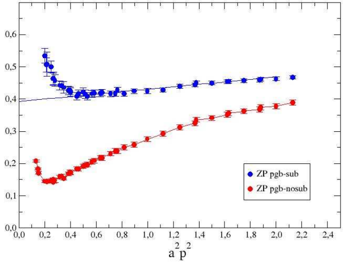
Goldstone pole

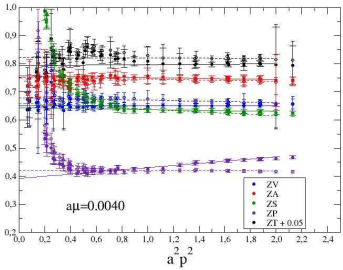
- the divergent term can be eliminated by (Giusti, Vladikas: 2000)

$$\Gamma_P^{\mathrm{SUB}}(p^2) = \frac{M_\pi^2(\mu_1)\Gamma_P(p^2,\mu_1) - M_\pi^2(\mu_2)\Gamma_P(p^2,\mu_2)}{M_\pi^2(\mu_1) - M_\pi^2(\mu_2)}$$



renormalization constants

$$Z_{\rm RGI}(g_0) = Z_{\rm MOM}(g_0, a\mu) C^{\rm MOM}(\mu)$$



$$Z_P^{\rm MOM}(\mu = 2GeV) = 0.39(1)(2)$$

PRELIMINARY

charm quark mass

- compute the D meson mass for a set of c bare quark masses, and a set of u,d quark masses
- the c quark is a "quenched quark" while the u,d quarks are fully dynamical

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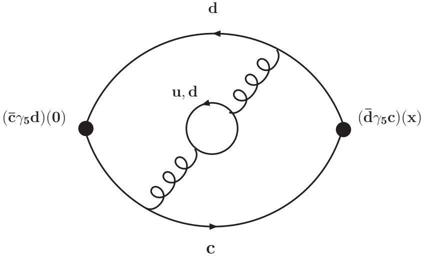

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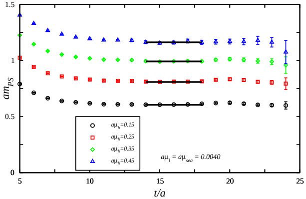
$$C(x_0) = a^3 \sum_{\mathbf{x}} \langle (\bar{c}\gamma_5 d)(0)\,(\bar{d}\gamma_5 c)(\mathbf{x}, x_0)\rangle$$



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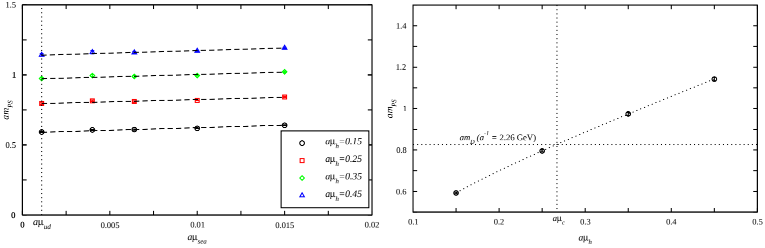

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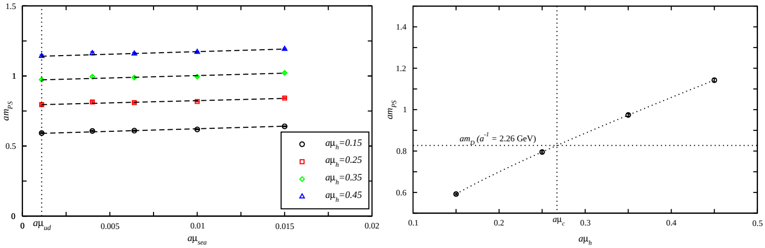
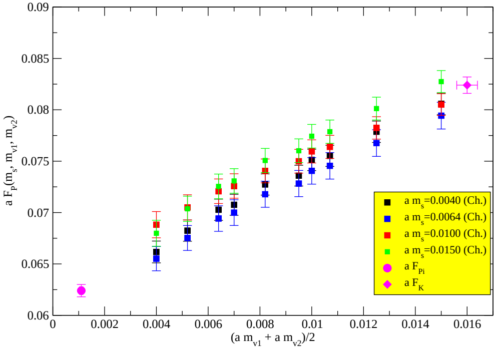


$$m_c^{\overline{\rm MS}}(\mu = 2GeV) = 1.23(4)(6) \quad \text{at} \quad a = 0.087 \ \text{fm}$$

PRELIMINARY

strange quark mass



$$m_s^{\overline{\rm MS}}(\mu = 2GeV) = 115(3)(5) \quad \text{at} \quad a = 0.087 \text{ fm}$$

PRELIMINARY

Conclusions

Summary:

- "is Wtm a way to go ?": I STILL THINK YES!
- it is possible to reach $m_{\mathrm{PS}} \sim 300$ MeV with stable simulations
- small statistical errors $\rightsquigarrow$ precise results for χPT parameters
- preliminary encoraguing results on renormalization constants and quark masses

To be done:

- complete check of systematic errors: continuum limit, finite volume effects

Perspectives:

- contact with phenomenology
- mixed action (sea: tmQCD; valence: overlap or OS) : *e.g.* B_K
- $N_{\mathrm{f}} = 2 + 1 + 1$ simulations are feasible

exceptional configurations

- Wilson fermions break chiral symmetry
- can be restored up to cutoff effects but...
- Wilson operator is not protected against zero modes
- functional integral over Grassmann variables cannot diverge
- after integration a small eigenvalue of the Wilson operator appears both in the quark propagator and in the fermionic determinant
- if I neglect the determinant (quenched model) the contribution of small eigenvalues in the correlators not balanced by the determinant
- large fluctuations in observables for certain gauge configurations $\rightarrow$ exceptional

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renormalization

- Wilson lattice QCD is renormalizable to all orders in perturbation theory (Reisz:1987-1988)
- counterterms to the action from the symmetries of the Wtm lattice action

$$\mathrm{tr}\{F_{\mu\nu}F_{\mu\nu}\}, \quad \overline{\chi}\chi, \quad m_0\overline{\chi}\chi, \quad i\mu_q\overline{\chi}\gamma_5\tau^3\chi$$

$$S_0 = S_G[A] + \int d^4x\overline{\chi}(x)\left[\gamma_\mu D_\mu + m_\mathrm{R} + i\mu_\mathrm{R}\gamma_5\tau^3\right]\chi(x)$$

$$g_\mathrm{R}^2 = g_0^2 Z_\mathrm{g}(g_0^2, a\mu)$$

$$m_\mathrm{R} = m_q Z_\mathrm{m}(g_0^2, a\mu) \qquad m_\mathrm{q} = m_0 - m_\mathrm{c}$$

$$\mu_\mathrm{R} = \mu_\mathrm{q} Z_\mu(g_0^2, a\mu),$$

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- counterterms to the action from the symmetries of the Wtm lattice action

$$\mathrm{tr}\{F_{\mu\nu}F_{\mu\nu}\}, \quad \bar{\chi}\chi, \quad m_0\bar{\chi}\chi, \quad i\mu_{\mathrm{q}}\bar{\chi}\gamma_5\tau^3\chi$$

$$S_0 = S_G[A] + \int d^4x\bar{\chi}(x)\left[\gamma_\mu D_\mu + m_{\mathrm{R}} + i\mu_{\mathrm{R}}\gamma_5\tau^3\right]\chi(x)$$

$$g_{\mathrm{R}}^2 = g_0^2 Z_{\mathrm{g}}(g_0^2, a\mu)$$

$$m_{\mathrm{R}} = m_q Z_{\mathrm{m}}(g_0^2, a\mu) \qquad m_{\mathrm{q}} = m_0 - m_{\mathrm{c}}$$

$$\mu_{\mathrm{R}} = \mu_{\mathrm{q}} Z_\mu(g_0^2, a\mu),$$

Symanzik expansion

- local effective theory to describe the long distance properties of Wtm close to the continuum limit (Symanzik:1981-1983)

$$S_{\mathrm{eff}} = S_0 + aS_1 + a^2 S_2 + \ldots \qquad S_k = \int d^4y \mathcal{L}_k(y)$$

- each term of the lagrangian has to be invariant under the symmetries of the lattice theory

$$\mathcal{O}_1 = i\overline{\chi}\sigma_{\mu\nu}F_{\mu\nu}\chi \quad \mathcal{O}_2 = m_{\mathrm{q}}\mathrm{tr}\{F_{\mu\nu}F_{\mu\nu}\} \quad \mathcal{O}_3 = m_{\mathrm{q}}^2\overline{\chi}\chi \quad \mathcal{O}_4 = m_{\mathrm{q}}\mu_{\mathrm{q}}i\overline{\chi}\gamma_5\tau^3\chi$$

$$\mathcal{O}_5 = \mu_{\mathrm{q}}^2\overline{\chi}\chi \quad \mathcal{O}_6 = m_{\mathrm{q}}\{\overline{\chi}\gamma_\mu\overrightarrow{D}_\mu\chi - \overline{\chi}\overleftarrow{D}_\mu\gamma_\mu\chi\} \quad \mathcal{O}_7 = \{\overline{\chi}\overrightarrow{D}_\mu\overrightarrow{D}_\mu\chi + \overline{\chi}\overleftarrow{D}_\mu\overleftarrow{D}_\mu\chi\}$$

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improvement

- improvement of the action (ALPHA:1996-)

$$S_{\rm impr}[\overline{\chi},\chi,U] = S[\overline{\chi},\chi,U] + \delta S[\overline{\chi},\chi,U] \quad \delta S[\overline{\chi},\chi,U] = a^5 \sum_x c_{\rm sw}\overline{\chi}(x)\frac{i}{4}\sigma_{\mu\nu}\hat{F}_{\mu\nu}(x)\chi(x)$$

- reparametrization of the bare parameters

$$g_{\rm R}^2 = \tilde{g}_0^2 Z_{\rm g}(\tilde{g}_0^2, a\mu) \qquad \tilde{g}_0^2 = g_0^2(1+b_{\rm g} am_{\rm q})$$

$$m_{\rm R} = \tilde{m}_q Z_{\rm m}(\tilde{g}_0^2, a\mu) \qquad \tilde{m}_q = m_q(1+b_{\rm m} am_{\rm q}) + \tilde{b}_{\rm m} a\mu_{\rm q}^2$$

$$\mu_{\rm R} = \tilde{\mu}_{\rm q} Z_\mu(\tilde{g}_0^2, a\mu) \qquad \mu_{\rm q}(1+b_\mu am_{\rm q})$$

- improvement of the operators

$$(A_{\rm R})_\mu^a = Z_{\rm A}(1+b_{\rm A} am_{\rm q})\left[A_\mu^a + ac_{\rm A}\partial_\mu P^a + a\mu_q\tilde{b}_{\rm A}\epsilon^{3ab}V_\mu^b\right]$$

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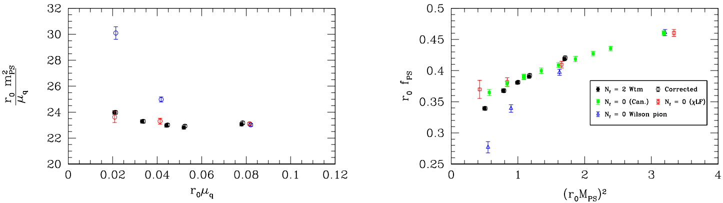
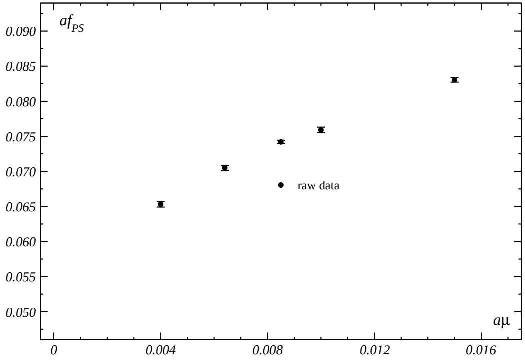
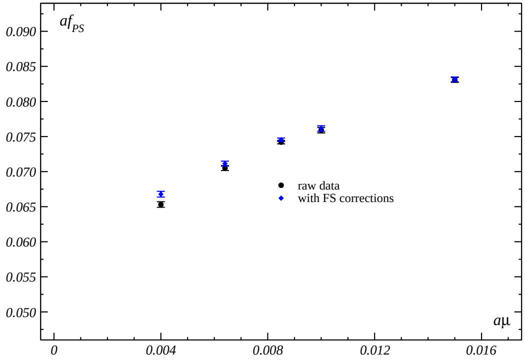
standard question on the bending



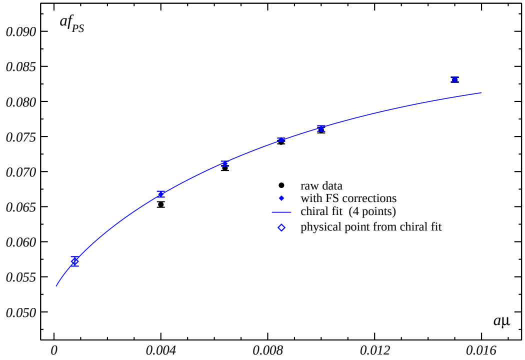
Pion sector: af_{PS} vs. $a\mu$ $\beta = 3.9$



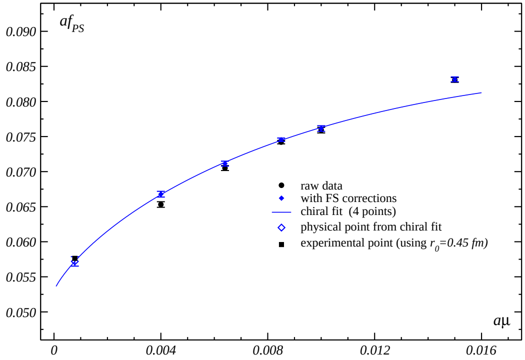
Pion sector: af_{PS} vs. $a\mu$ $\beta = 3.9$



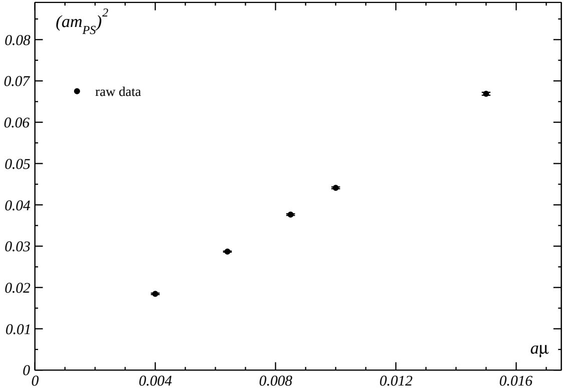
Pion sector: af_{PS} vs. $a\mu$ $\beta = 3.9$



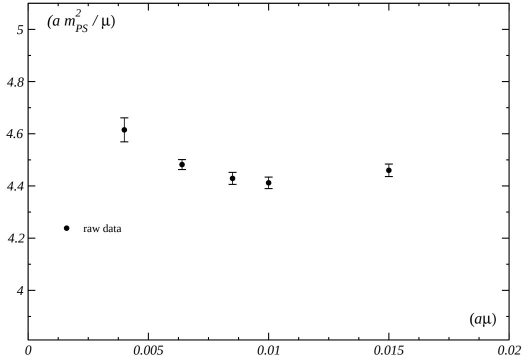
Pion sector: af_{PS} vs. $a\mu_q$ $\beta = 3.9$



Pion sector: $(am_{PS})^2$ vs. $a\mu$ $\beta = 3.9$

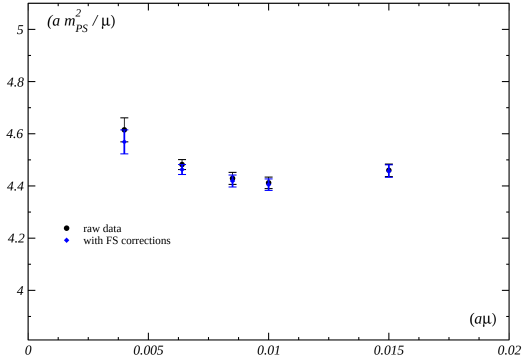


Pion sector: m_{PS}^2/μ vs. μ $\beta = 3.9$



Pion sector: m^2_{PS}/μ vs. μ

$\beta = 3.9$



Choice of the gauge action

- Wilson-type fermions (plain and twisted) have a non-trivial phase structure at finite lattice spacing (Aoki; Sharpe, Singleton)
- The strength of the phase transition depends on details of the action
  - ⋆ gluonic: b_1
  - ⋆ fermionic: $c_{\rm sw}$
- tree-level Symanzik improved (tlSym) gauge action

$$S_g = \frac{\beta}{3} \sum_x \left[(1 - 8b_1) \sum_{\mu<\nu}^{4} \left(1 - \mathrm{ReTr}\left(U^{1\times 1}_{x,\mu,\nu} \right) \right) + b_1 \sum_{\mu\neq\nu}^{4} \left(1 - \mathrm{ReTr}\left(U^{1\times 2}_{x,\mu,\nu} \right) \right) \right]$$

with $b_1 = -1/12$

- tlSym:
  - ⋆ weakens the first order phase transitions compared to Wilson gauge action ($b_1 = 0$)
  - ⋆ better scaling than DBW2 ($b_1 = -1.4088$)

(Farchioni *et. al.*, 2004-2005)

- Consequence of first order phase transitions:
  - ⋆ For a given a, simulation is safe if $\mu > \mu_{\rm end-point} \sim a^2\Lambda^3_{\rm QCD}$
  - ⋆ For a given value of $m_{\rm PS}$ one can find a lattice spacing $a_{\rm max}$ such that simulations at $a < a_{\rm max}$ can be safely performed

Algorithm: speeding-up the HMC Wilson fermions

- Variant of the HMC algorithm

  (C. Urbach, K. Jansen, A. Shindler, U. Wenger, 2005)

  - ⋆ even/odd preconditioning
  - ⋆ mass preconditioning (Hasenbusch, 2001)
  - ⋆ multiple time scale integration

- Other variants: all of them are efficient to reach small quark masses

  - ⋆ domain decomposition (Lüscher, 2003-2004)
  - ⋆ RHMC (Clark, Kennedy, 2003)
  - ⋆ QCDSF collab. (2003)

- Wilson fermions are back in the game

Simulations: plan

- ⋆ fermion: $N_{\mathrm{f}} = 2$ maximally twisted mass QCD
- ⋆ gauge: tlSym
- ⋆ three lattice spacings: $0.075 - 0.115$ fm
- ⋆ $270 \lesssim m_{\mathrm{PS}} \lesssim 550$ MeV
- ⋆ $L > 2$ fm

Simulations: setup

β	target a (fm)	$L^3 \cdot T$	$\kappa_{\rm crit}$	$a\mu$	$N_{\rm traj}$	target $m_{\rm PS}$ (MeV)
4.05	~ 0.075	$32^3 \cdot 64$	0.15701	0.0030	5000	~ 270
				0.0060	5000	~ 380
				0.0080	3000	~ 430
				0.0120	2000	~ 530
3.9	~ 0.09	$24^3 \cdot 48$	0.160856	0.0040	9400	~ 300
				0.0064	5000	~ 380
				0.0085	5000	~ 440
				0.0100	5000	~ 480
				0.0150	5000	~ 580
		$32^3 \cdot 64$		0.0040	5000	~ 300
3.8	~ 0.11	$20^3 \cdot 48$	0.164099	0.0060	3900	~ 290
				0.0090	3600	~ 390
				0.0120	5000	~ 450
				0.0150	3300	~ 510

Monte Carlo histories of plaquette

Quark masses twisted mass fermions

Aachen - 22.02.07

A. Shindler

Intro

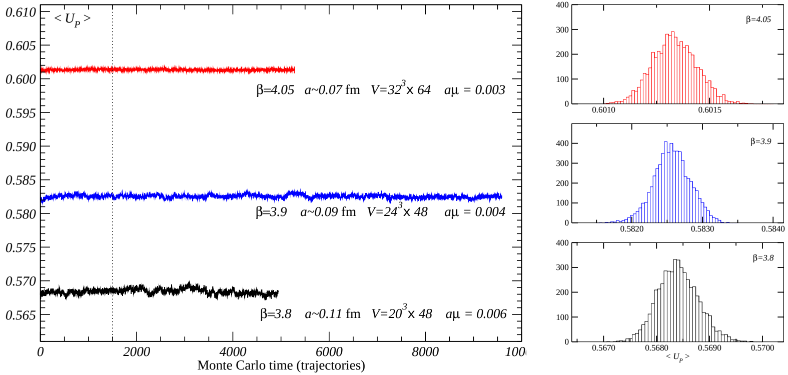
tmQCD

Simulations

Renorm.

Quark masses

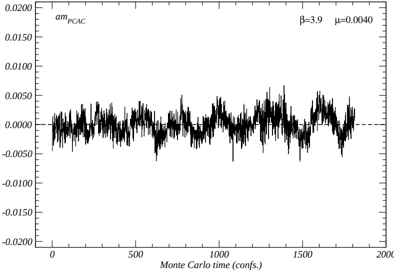
Conclusions

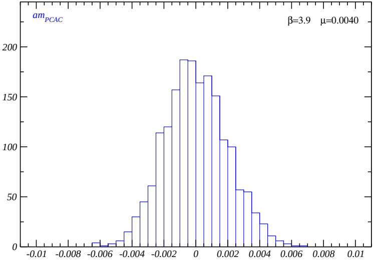


Autocorrelations $\beta = 3.9$

- plaquette : $\tau_{\rm int}(P) \in [10 - 55]$ (in units of $\tau = 0.5$)
- $f_{\rm PS}$: $\tau_{\rm int}(af_{\rm PS}) \in [4 - 7]$
- configurations saved every 2 trajectories
- ILDG

Monte Carlo history of m_{PCAC} $\beta = 3.9$

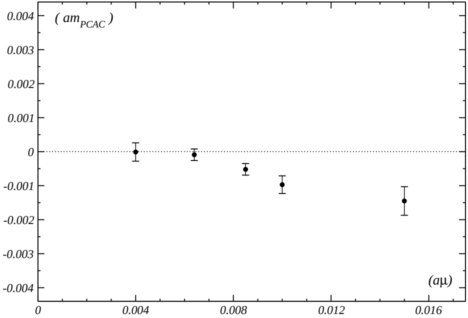




- $V = 24^3 \cdot 48, \quad \beta = 3.9, \quad a\mu = a\mu_{\mathrm{min}} = 0.004$

$$am_{\mathrm{PCAC}}(a\mu = a\mu_{\mathrm{min}}) = 0 \pm \mathcal{O}((a\mu_{\mathrm{min}})(a\Lambda_{\mathrm{QCD}}))$$

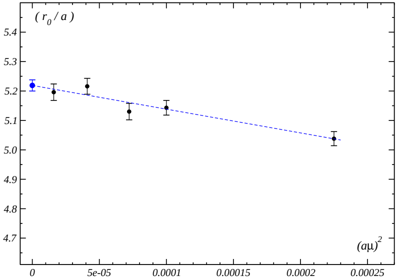
am_{PCAC} vs. $a\mu$ $\beta = 3.9$



- $am_{\mathrm{PCAC}}(a\mu_{\mathrm{min}}) = -0.00001(27)$
- for all μ values: $am_{\mathrm{PCAC}} \lesssim (a\mu)(a\Lambda_{\mathrm{QCD}})$
- The weak μ-dependence of m_{PCAC} is an $\mathcal{O}(a)$ effect
  $\rightsquigarrow$ $\mathcal{O}(a^2)$ artifacts in physical quantities

Setting the scale: r_0 vs. μ^2 $\beta = 3.9$

- Sommer parameter r_0: static inter-quark force



- dependence on μ^2
- good accuracy: $r_0/a = 5.22(2)$
- setting the scale: use several quantities, e.g. $m_\pi\,,\ f_\pi\,,\ m_K\,,\ m_{K*}\,,\ f_K\,,\ m_N, \ldots$

Pion sector: mass splitting

- flavour symmetry breaking generates mass splittings
- mass splitting between charged and neutral pions expected to be larger than for heavier hadrons
- the effect should vanish in the continuum limit:

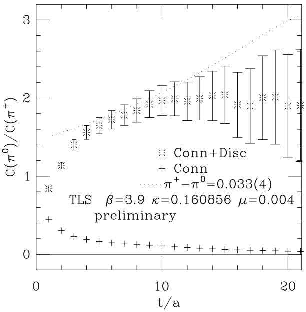
$$(m_{\rm PS}^{0})^2 - (m_{\rm PS}^{\pm})^2 = \mathcal{O}(a^2)$$

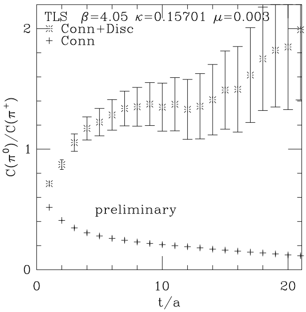
Disconnected correlations needed for π^0 meson:

- stochastic volume sources: 24 sources per gauge config
- measurements separated by 20 trajectories

Pion mass splitting $\beta = 3.9$ and 4.05

PRELIMINARY





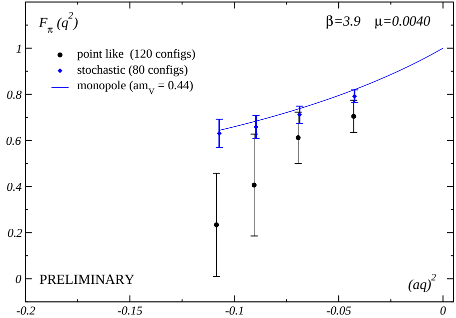
- at $\beta = 3.9$: π^0 is 20% lighter than $\pi^{\pm}$
- $r_0^2((m_{\rm PS}^0)^2 - (m_{\rm PS}^{\pm})^2) = c(a/r_0)^2$ with $c = -4.5(1.8)$

  2 times smaller than in quenched and opposite sign
- smaller effect at $\beta = 4.05$
- for the vector meson the splitting is compatible with zero

Pion form factor

$\beta = 3.9$

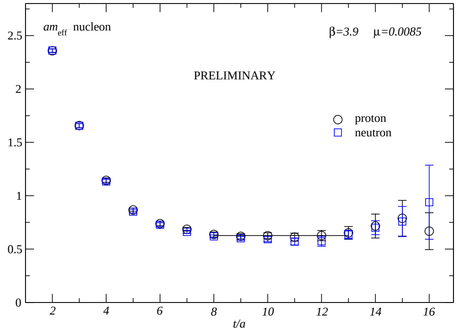
PRELIMINARY



- at $\beta = 3.9 \quad a\mu = 0.004 \quad m_{\mathrm{PS}} \sim 300$ MeV
- stochastic propagators with θ boundary conditions

Nucleon : effective mass $\beta = 3.9$

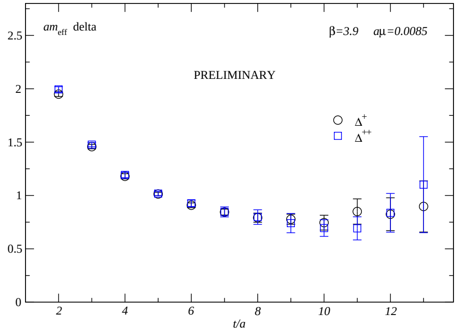
PRELIMINARY



- at $\beta = 3.9$ $a\mu = 0.0085$ $m_{PS} \sim 400$ MeV $m_p \sim 1.4$ GeV $L \sim 2$ fm
- 80 confs. separated by 40 trajectories each
- point like sources, randomly located
- $m_P = m_N$

Δ : effective mass $\beta = 3.9$

PRELIMINARY



- at $\beta = 3.9$ $a\mu = 0.0085$ $m_{\rm PS} \sim 400$ MeV $m_{\Delta^+} \sim 1.8$ GeV $L \sim 2$ fm
- 50 confs. separated by 60 trajectories each
- splitting between m_{Δ^+} and $m_{\Delta^{++}}$ is not observed