$m_{\rm b}^{\overline{\rm DR}}(\mu)$ to high accuracy

Luminita Mihaila

Institut für Theoretische Teilchenphysik

Universität Karlsruhe

in collaboration with R. Harlander[(a)], M. Steinhauser[(b)]

(a) Fachbereich C, Theoretische Physik, Universität Wuppertal

(b) Institut für Theoretische Teilchenphysik, Universität Karlsruhe

Outline

- Motivation
- Dimensional Reduction Framework
- Phenomenological analysis: $m_{\mathrm{b}}^{\overline{\mathrm{DR}}}(\mu)$ to 4-loop accuracy
- Conclusions

Motivation

- Precision calculations of the LHC- and ILC-observables &
  - Renormalization of SUSY Gauge Theories
  - manifestly SUSY and gauge invariant Regularization

Motivation

- Precision calculations of the LHC- and ILC-observables &
  - Renormalization of SUSY Gauge Theories
  - manifestly SUSY and gauge invariant Regularization

  Is DRED a possible solution ?

- Removal of DRED inconsistencies [W. Siegel '80], [D. Stöckinger '05]
  - possible SUSY violation at HO

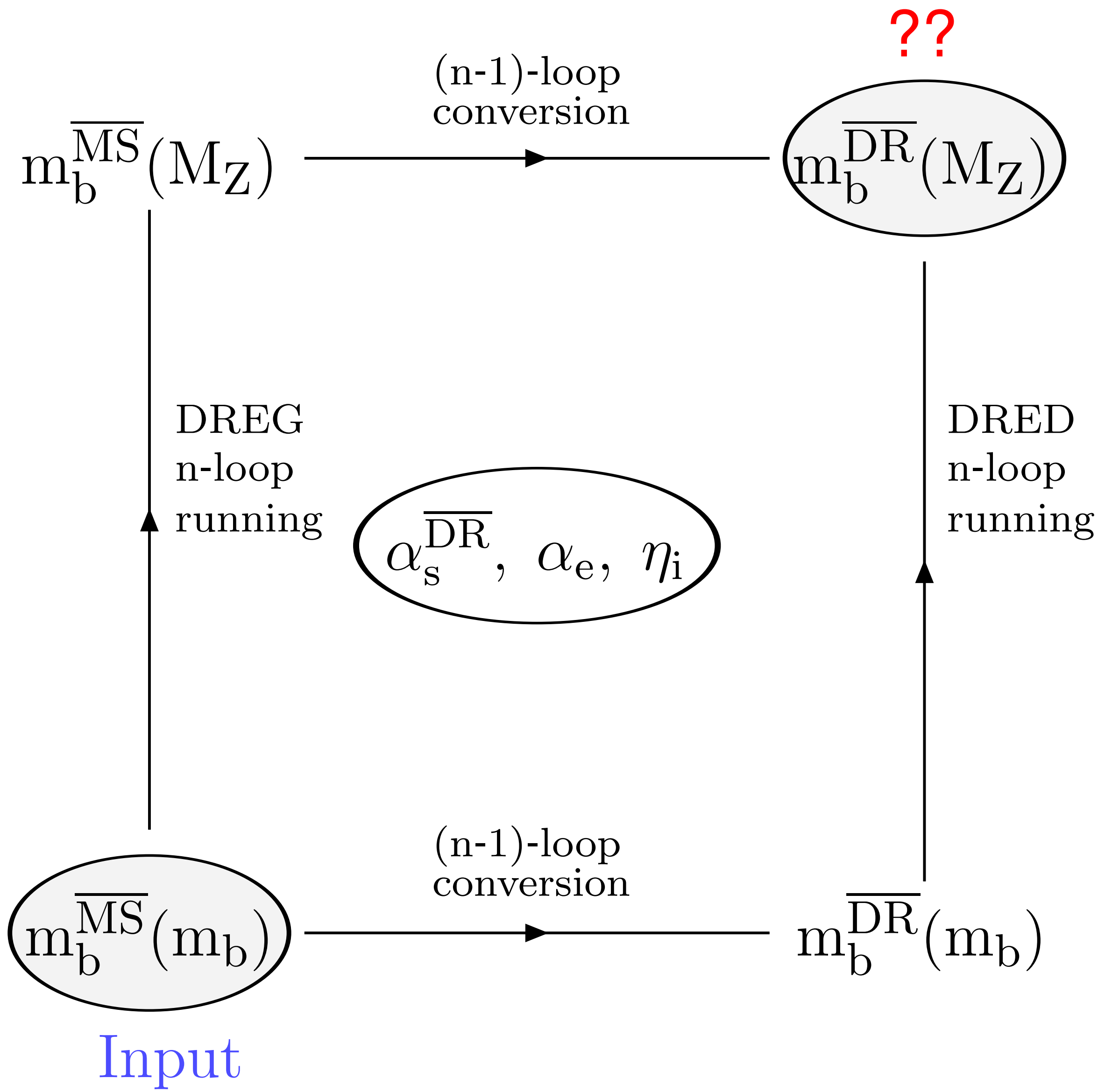
    [L. Avdeev, G. Chochia, A. Vladimirov '81], [I. Jack and D. R. T. Jones '97]
  - SUSY preserved in all present 1- and 2-loop checks

    [W. Hollik and D. Stöckinger '05]
  - Factorization Theorem within DRED proved

    [A. Signer and D. Stöckinger '05]

Motivation

- Precision calculations of the LHC- and ILC-observables &
  - Renormalization of SUSY Gauge Theories
  - manifestly SUSY and gauge invariant Regularization
- Running analysis relating low energy observables to their values at the GUT scale.
  - $m_{\mathrm{b}}^{\overline{\mathrm{DR}}}(M_Z)$ input parameter for all SUSY models with Yukawa-coupling unification
  - accuracy on $m_{\mathrm{b}}^{\overline{\mathrm{DR}}}(M_Z)$ translates into accuracy within which Yukawa-couplings unify
  - same accuracy as for $m_{\mathrm{b}}^{\overline{\mathrm{MS}}}(M_Z)$ is desirable



Aim : check 2 ways at n = 1, 2, 3, 4-loops

Framework

Quasi-4-dim. space (Q4S): $4 = d \oplus 4 - d$

- *Quasi*-4-dim metric tensor: $G_{\mu\nu} = g_{\mu\nu} + \tilde{g}_{\mu\nu}$
- Dirac matrices in Q4S: $\Gamma_\mu = \gamma_\mu + \tilde{\gamma}_\mu$
- space-time coordinates continued from 4 to $d \leq 4$ dim.
- the number of field components unchanged

Framework

Quasi-4-dim. space (Q4S): $4 = d \oplus 4 - d$

- *Quasi*-4-dim metric tensor: $G_{\mu\nu} = g_{\mu\nu} + \tilde{g}_{\mu\nu}$
- Dirac matrices in Q4S: $\Gamma_\mu = \gamma_\mu + \tilde{\gamma}_\mu$
- space-time coordinates continued from 4 to $d \leq 4$ dim.
- the number of field components unchanged
  - 4-dim gluon field: $A^a_\mu = V^a_\mu + S^a_\mu\,,$

$$\begin{aligned} V^a_\mu &= g_{\mu\nu}\, A^a_\nu = \quad d\text{- dim. vector} \\ S^a_\mu &= \tilde{g}_{\mu\nu}\, A^a_\nu = \quad \varepsilon \text{ scalar} \end{aligned}$$

under gauge transformations

Renormalization

$$\mathcal{L}_\mathrm{B} = \mathcal{L}_\mathrm{B}^d + \mathcal{L}_\mathrm{B}^\varepsilon$$

- $\mathcal{L}_\mathrm{B}^d$ same as in DREG
- $\mathcal{L}_\mathrm{B}^\varepsilon$ new contribution due to ε-scalars

$$\mathcal{L}_\mathrm{B}^d = -\frac{1}{4}G^{a,ij}G^a_{ij} - \frac{(\partial^i V_i^a)^2}{2(1-\xi)} + \mathcal{L}^d_{\mathrm{ghost,B}} + i\bar{\psi}^\alpha \gamma^i D_i^{\alpha\beta}\psi^\beta$$

$$\mathcal{L}_\mathrm{B}^\varepsilon = \frac{1}{2}(D_i^{ab}S_\sigma^b)^2 - g\bar{\psi}\tilde{\gamma}_\sigma T^a \psi S_\sigma^a - \frac{1}{4}g^2 f^{abc}f^{ade}S_\sigma^b S_{\sigma'}^c S_\sigma^d S_{\sigma'}^e$$

- each term in $\mathcal{L}_\mathrm{B}^\varepsilon$ invariant under gauge transformations
  - no reason that Yukawa-type $\bar{\psi}\psi S$ and $\bar{\psi}\psi V$ vertices renormalize the same way [except for SUSY theories !]
  - $f - f$ structure not preserved under renormalization

Renormalization(2)

$$
\begin{aligned}
\mathcal{L}^{\varepsilon} &= \frac{1}{2} Z_3^{\varepsilon} (\partial_i S_\sigma)^2 + Z^{\varepsilon\varepsilon V} g f^{abc} \partial_i S_\sigma^a V^{b,i} S_\sigma^c \\
&+ Z^{\varepsilon\varepsilon VV} g^2 f^{abc} f^{ade} V_i^b S_\sigma^c V^{d,i} S_\sigma^e - Z_1^{\varepsilon} g_e \bar{\psi} T^a \tilde{\gamma}^{\sigma} \psi S_\sigma^a \\
&- \frac{1}{4} \sum_{r=1}^{p} Z_{1,r}^{4\varepsilon} \eta_r H_r^{abcd} S_\sigma^a S_{\sigma'}^c S_\sigma^b S_{\sigma'}^d
\end{aligned}
$$

- Evanescent Yukawa-type g_e and $\boldsymbol{p}$ quartic couplings η_r
- a possible choice of H^{abcd} for $SU(3)$ case

$$
H_1 = \frac{1}{2} \left(f^{ace} f^{bde} + f^{ade} f^{bce} \right)
$$

$$
H_2 = \frac{1}{2} \delta^{ab} \delta^{cd} \qquad H_3 = \frac{1}{2} (\delta^{ac} \delta^{bd} + \delta^{ad} \delta^{bc})
$$

QCD β- and γ_m-functions within DRED

- Dimensional Reduction $\oplus$ Minimal Subtraction $\overline{\text{DR}}$

$$\beta_s^{\overline{\text{DR}}}(\alpha_s^{\overline{\text{DR}}}, \alpha_e, \{\eta_r\}) = -\sum_{i,j,k,l,m} \beta_{ijklm}^{\overline{\text{DR}}} \left(\frac{\alpha_s^{\overline{\text{DR}}}}{\pi}\right)^i \left(\frac{\alpha_e}{\pi}\right)^j \left(\frac{\eta_1}{\pi}\right)^k \left(\frac{\eta_2}{\pi}\right)^l \left(\frac{\eta_3}{\pi}\right)^m$$

$$\beta_e(\alpha_s^{\overline{\text{DR}}}, \alpha_e, \{\eta_r\}) = -\sum_{i,j,k,l,m} \beta_{ijklm}^{e} \left(\frac{\alpha_s^{\overline{\text{DR}}}}{\pi}\right)^i \left(\frac{\alpha_e}{\pi}\right)^j \left(\frac{\eta_1}{\pi}\right)^k \left(\frac{\eta_2}{\pi}\right)^l \left(\frac{\eta_3}{\pi}\right)^m$$

$$\beta_{\eta_r}(\alpha_s^{\overline{\text{DR}}}, \alpha_e, \{\eta_r\}) = -\sum_{i,j,k,l,m} \beta_{ijklm}^{\eta_r} \left(\frac{\alpha_s^{\overline{\text{DR}}}}{\pi}\right)^i \left(\frac{\alpha_e}{\pi}\right)^j \left(\frac{\eta_1}{\pi}\right)^k \left(\frac{\eta_2}{\pi}\right)^l \left(\frac{\eta_3}{\pi}\right)^m$$

$$\gamma_m^{\overline{\text{DR}}}(\alpha_s^{\overline{\text{DR}}}, \alpha_e, \{\eta_r\}) = -\sum_{i,j,k,l,m} \gamma_{ijklm}^{\overline{\text{DR}}} \left(\frac{\alpha_s^{\overline{\text{DR}}}}{\pi}\right)^i \left(\frac{\alpha_e}{\pi}\right)^j \left(\frac{\eta_1}{\pi}\right)^k \left(\frac{\eta_2}{\pi}\right)^l \left(\frac{\eta_3}{\pi}\right)^m$$

QCD and $\overline{\text{DR}}$-scheme

- $\beta_s^{\overline{\text{DR}}}$:
  - 1- and 2-loop [S. P Martin and M. Vaughn '83]
  - 3-loop [Z. Bern et al '02], [R. Harlander, P. Kant, L. M., M. Steinhauser '06]
  - 4-loop [[R. Harlander, D. R. T. Jones, P. Kant, L. M., M. Steinhauser '06]
- $\gamma_m^{\overline{\text{DR}}}$:
  - 1-loop [S. P Martin and M. Vaughn '83]
  - 2-loop [L. Avdeev and Y. Kalmikov '97]
  - 3-loop [[R. Harlander, P. Kant, L. M., M. Steinhauser '06]
  - 4-loop [R. Harlander, D. R. T. Jones, P. Kant, L. M., M. Steinhauser '06]
- β_e :
  - 1-loop [L. Avdeev and Y. Kalmikov '97], [I. Jack and D. R. T. Jones '97]
  - 2- and 3-loop [R. Harlander et al '06]
- β_{η_r} :
  - 1-loop [I. Jack and D. R. T. Jones '79 unpublished],[R. Harlander et al '06]

Relation $m^{\overline{\mathrm{DR}}} \leftrightarrow m^{\overline{\mathrm{MS}}}$

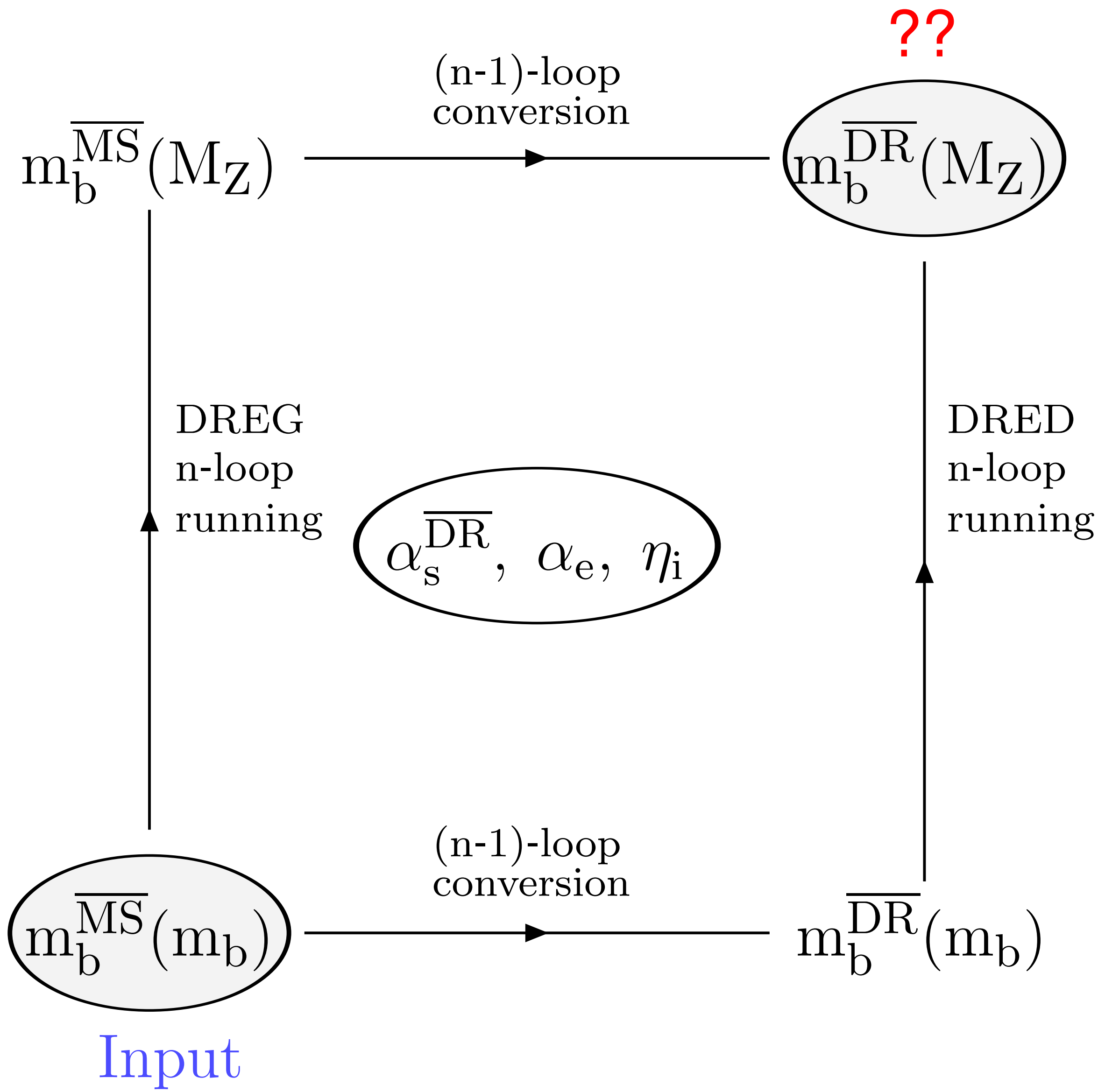
- Extract $m_{\mathrm{b}}^{\overline{\mathrm{DR}}}(M_Z)$ from accurately det. $m_{\mathrm{b}}^{\overline{\mathrm{MS}}}(m_b)$

$$m_b^{\overline{\mathrm{DR}}}(\mu) = m_b^{\overline{\mathrm{MS}}}(\mu)\Big[1 + \delta_m^{(1l)}(\alpha_e) + \delta_m^{(2l)}(\alpha_s^{\overline{\mathrm{MS}}}, \alpha_e) + \delta_m^{(3l)}(\alpha_s^{\overline{\mathrm{MS}}}, \alpha_e, \eta_i)\Big]\Big|_{\mu=\mu_S},$$

$\{\alpha_s^{\overline{\mathrm{DR}}}, \alpha_e, \eta_i\}\big|_{\mu=\mu_S}$ have to be known.

  - Log contributions absent (mass-independent schemes)
  - very good convergence of the perturbative series

- Evolve $m_b^{\overline{\mathrm{DR}}}(\mu)$ in $\overline{\mathrm{DR}}$ from $\mu = \mu_S$ to $\mu = M_Z, M_{\mathrm{SUSY}}, \cdots$
- 2-step approach for computing $\boldsymbol{m}_{\mathrm{b}}^{\overline{\mathrm{DR}}}(M_Z)$ [H. Baer et al '02]:



Aim : check 2 ways at $n = 1, 2, 3, 4$-loops

Computation of $\overline{\mathrm{DR}}$ coupling constants

Input parameter $\alpha_s^{\overline{\mathrm{MS}},(5)}(M_Z)$

- SUSY-QCD ($\hat{=}$ full) : only one coupling $\alpha_s^{\overline{\mathrm{DR}},(\mathrm{full})}(\mu)$
  $\beta_s = \beta_e = \beta_{\eta_1}$ and $\beta_{\eta_2} = \beta_{\eta_3} = 0$
- QCD(5 flavours) as low-energy effective theory of SUSY-QCD
  $\Rightarrow$ integrate out all SUSY-particles and top-quark at $\mu = \boldsymbol{\mu}_{\mathrm{dec}}$

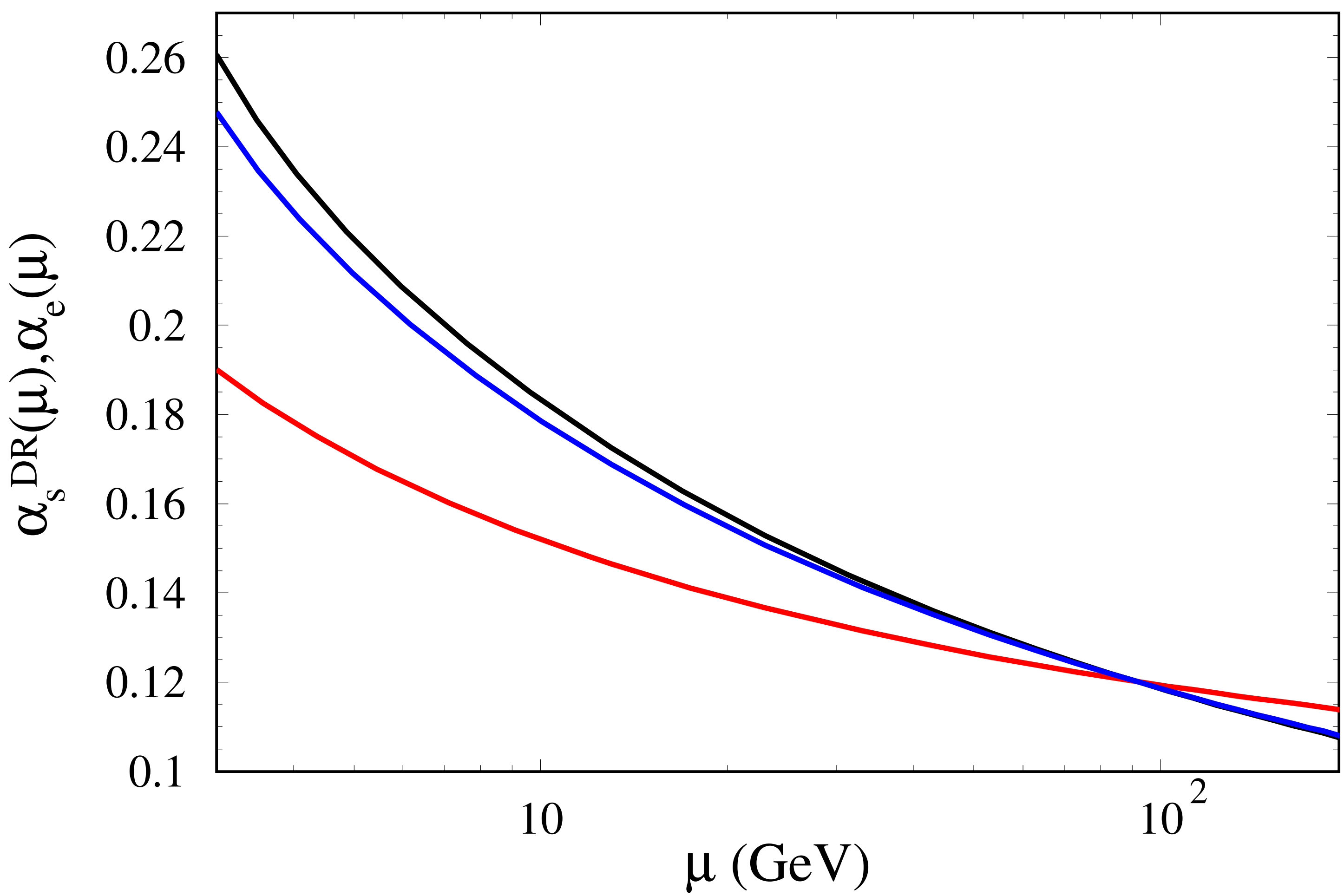
$$\alpha_e^{(\mathrm{full})}(\mu_{\mathrm{dec}}) = \alpha_s^{\overline{\mathrm{DR}},(\mathrm{full})}(\mu_{\mathrm{dec}}) = \eta_1^{(\mathrm{full})}(\mu_{\mathrm{dec}})\,,$$
$$\eta_2^{(\mathrm{full})}(\mu_{\mathrm{dec}}) = \eta_3^{(\mathrm{full})}(\mu_{\mathrm{dec}}) = 0$$

- Extract $\{\alpha_s^{\overline{\mathrm{DR}},(5)}, \alpha_e^{(5)}, \eta_i^{(5)}\}\big|_{\mu=\mu_{\mathrm{dec}}}$ from conversion relation $\overline{\mathrm{MS}} \to \overline{\mathrm{DR}}$

$$\alpha_s^{\overline{\mathrm{DR}},(5)}(\mu) = \alpha_s^{\overline{\mathrm{MS}},(5)}(\mu)\Big[1 + \delta_s^{(1l)}(\alpha_s^{\overline{\mathrm{MS}}}) + \delta_s^{(2l)}(\alpha_s^{\overline{\mathrm{MS}},(5)}, \alpha_e^{(5)}) + \delta_s^{(3l)}(\alpha_s^{\overline{\mathrm{MS}},(5)}, \alpha_e^{(5)}, \eta_i^{(5)})\Big]\Big|_{\mu=\mu_{dec}}$$

- Evolve $\{\alpha_s^{\overline{\mathrm{DR}}}, \alpha_e, \eta_i\}$ within $\overline{\mathrm{DR}}$ from μ_{dec} to $\mu_S = m_b, M_Z, \cdots$

- $\mu_{\rm dec} = M_Z$
- 4-, 3- and 1-loop running for $\alpha_s^{\overline{\rm DR}}(\mu)$ (black), α_e (red), and η_1 (blue)



Phenomenological analysis

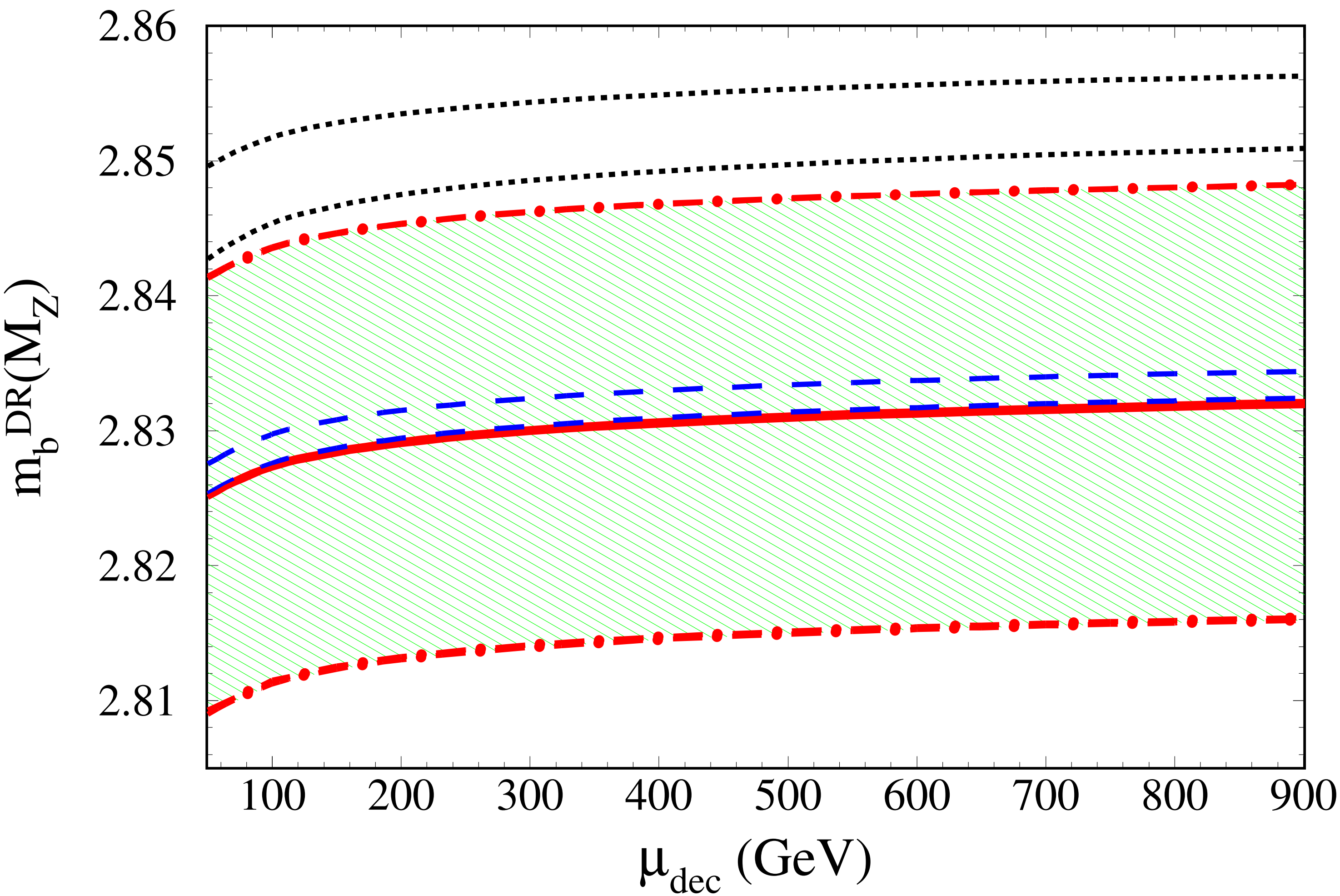
- non-SUSY theories: $\alpha_s^{\overline{\rm DR}} \neq \alpha_e \neq \eta_1$ and $\eta_2 \neq \eta_3 \neq 0$ essential
- Numerical example
  - Integrate out all SUSY particles at $\mu = M_Z \Rightarrow$
    $\alpha_s^{\overline{\rm DR}}(M_Z) = \alpha_e(M_Z) = \eta_1(M_Z) = 0.1202\,,$
    $\eta_2(M_Z) = \eta_3(M_Z) = 0$
  - Evolve $\{\alpha_s^{\overline{\rm DR}}, \alpha_e, \eta_i\}$ down to $\mu_b = 4.2$ GeV
    $\alpha_s^{\overline{\rm DR}}(\mu_b) = 0.2335\,, \quad \alpha_e(\mu_b) = 0.1767\,, \quad \eta_1(\mu_b) = 0.2234$
    $\eta_2(\mu_b) = -0.0206\,, \quad \eta_2(\mu_b) = -0.0071$
    Evaluate $m_b^{\overline{\rm DR}}(\mu_b) = 4.117$ GeV
  - If $\alpha_s^{\overline{\rm DR}} = \alpha_e = \eta_1 = 0.2335\,, \eta_2 = \eta_3 = 0$
    Evaluate $m_b^{\overline{\rm DR}}(\mu_b) = 4.090$ GeV

$$\Rightarrow \quad \delta m_b = 27 \text{ MeV !}$$

Input parameters:

$\alpha_s^{\overline{\mathrm{MS}}}(M_Z) = 0.1189 \pm 0.001$ [S. Bethke '06]

$m_b^{\overline{\mathrm{MS}}}(m_b) = 4.164 \pm 0.025$ GeV [J. H. Kühn, M. Steinhauser, C. Sturm '07]



- 2-loop running + 1-loop conversion $\overline{\mathrm{MS}} \to \overline{\mathrm{DR}}$
  theoretical uncertainty $=22$ MeV $\simeq$ exp. error $=20$ MeV
- 3-loop running + 2-loop conversion $\overline{\mathrm{MS}} \to \overline{\mathrm{DR}}$
  reduced theoretical uncertainty ($\simeq 1$ MeV)
- If consistently evaluated, the numerical difference between path (1) */vs/* path $(1')$ $\simeq 6\,,\ 1\,,\ 0.1$ MeV for 2-,3-,4-loop running
- If evanescent couplings identified with $\alpha_s^{\overline{\mathrm{DR}}}$,
  the numerical difference path (1) */vs/* path $(1')$ $\simeq 10$ MeV
  for a mixed 3($\overline{\mathrm{MS}}$)- and 2($\overline{\mathrm{DR}}$)-loop running [H. Baer et al '02]

Conclusions

- A consistent approach to compute $\boldsymbol{m}_{\mathrm{b}}^{\overline{\mathrm{DR}}}(\mu)$ with 4-loop accuracy is proposed
- At least 3-loop accuracy required for high-precision analysis
- Correct treatment of the evanescent couplings essential
- Very good convergence of the perturbative series $m^{\overline{\mathrm{DR}}}(\mu) \leftrightarrow m^{\overline{\mathrm{MS}}}(\mu)$