Accurate Determinations of α_s from Realistic Lattice QCD

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α_s from Lattice QCD (LQCD)

- Lattice QCD = most complete implementation of QCD.
  - Lattice = ultraviolet regulator.
  - Implements nonperturbative QCD.
  - Implements perturbative QCD.

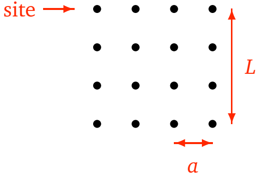
- Tune LQCD parameters to experimental data ⇒ can extract α_s by "measuring" short-distance quantities in simulations.

- Capable of very high precision.
  - Experimental inputs are hadron masses (for tuning).
  - Great latitude to design many short-distance quantities that are free of systematic errors (nonperturbative contamination) and easy to measure in simulations.

What is Lattice QCD?

Lattice Approximation

Continuous
Space & Time $\longrightarrow$



$\Rightarrow$ Fields $\psi(x)$, $A_\mu(x)$ specified only at grid sites; interpolate for other points.

K. Wilson (1974)

$\Rightarrow$ QCD $\rightarrow$ multidimensional integration.

$$\int \mathscr{D}A_\mu \ldots \mathrm{e}^{-\int L\mathrm{d}t} \longrightarrow \int \prod_{x_j \epsilon \, \mathrm{grid}} \mathrm{d}A_\mu(x_j) \ldots \mathrm{e}^{-a \sum L_j}.$$

$\Rightarrow$ Millions of integration variables.

$\Rightarrow$ Numerical Monte Carlo integration.

$\Rightarrow$ Integrate quarks out $\rightarrow \det(D \cdot \gamma + m)$ (expensive!).

Two QCD Breakthroughs

1) Larger a (1992)

Before $\Rightarrow$ need $a < 0.05\,\mathrm{fm}$.

Now $\Rightarrow$ $a = 0.1$–$0.4\,\mathrm{fm}$ works.

Simulation cost $\propto (1/a)^6$

$\Rightarrow$ new simulations cost 10^2–10^6 times less!

2) Smaller u/d Quark Masses (2000)

Before $\Rightarrow$ $m_{u/d}$ 10–20$\times$ too big; vac. pol'n impossible.

Now $\Rightarrow$ 3–5$\times$ smaller masses; extrapolate to real QCD.

Vac. pol'n enters at 15–30%

$\Rightarrow$ high-precision (few %) simulations possible now, for first time.

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Tuning and Testing Lattice QCD

Tuning QCD Parameters

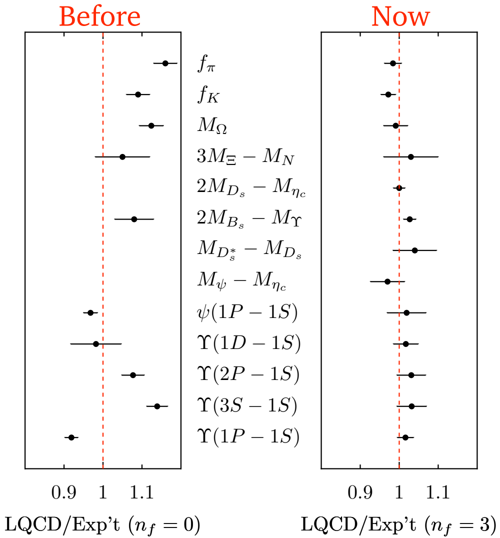
- Tune 5 free parameters (bare $m_u = m_d$, m_s, m_c, m_b and α_s) to match experimental data for m_π^2, $2m_K^2 - m_\pi^2$, m_{η_c}, m_Υ, and $m_{\Upsilon'} - m_\Upsilon$.
  - Each quantity depends strongly on only one parameter, weakly on others.
  - Experimental errors negligible.
  - Residual isospin/electromagnetic errors negligible.
  - Bare α_s replaced by lattice spacing a as tunable parameter: choose bare α_s; tune a to corresponding value.

- No other free parameters.

- Test by: a) computing lots of other quantities and comparing with experiment; b) symmetry restoration.

Davies et al, Phys. Rev. Lett. 92:022001, 2004. (HPQCD, MILC, Fermilab, UKQCD)

Lattice QCD/Experiment (no free parameters!):



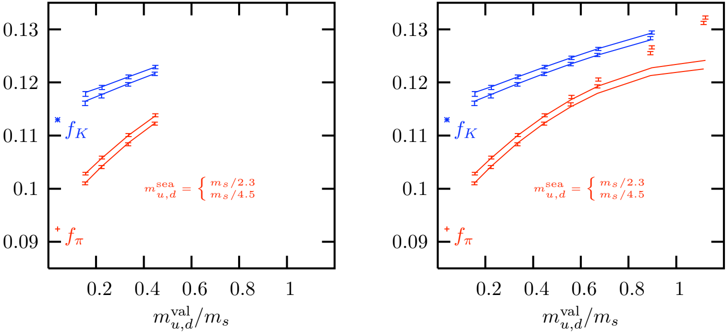
Tests:

- $m_{u,d}$ extrapolation;
- masses and wavefunctions;
- s quark;
- light-quark baryons;
- light-heavy mesons;
- heavy quarks (no potential model. . .);
- improved staggered quark vacuum polarization.

⇒ Most accurate strong interaction calculation in history!

Example Analysis:

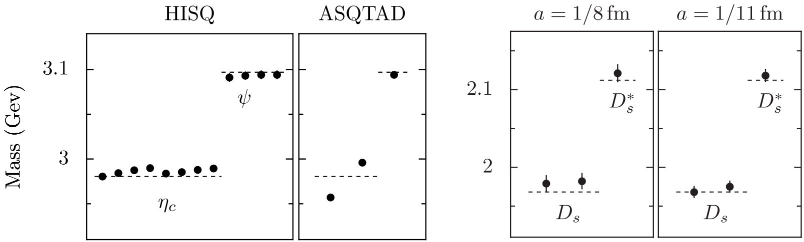
f_π and f_K fits versus valence u, d mass:



N.B. Quark mass problem solved!

Example Analysis:

Hyperfine mass splittings for mesons with charmed quarks using HISQ action:



N.B. Few MeV precision with no free parameters!

Follana et al (2007).

Test Discretization — Symmetry Restoration

High precision + large $a \Rightarrow$ improved discretizations: eg,

$$\frac{\partial\psi(x_j)}{\partial x} = \Delta_x\psi(x_j) - c(a)\,\frac{a^2}{6}\,\Delta_x^3\psi + \mathcal{O}(a^4)$$

$$\frac{\psi(x_j+a)-\psi(x_j-a)}{2a}$$

Finite-a correction.

where (from perturbation theory)

$$c(a) = 1 + c_1\,\alpha_s(\pi/a) + \cdots$$

Numerical Analysis

Mimics effects of $p > \pi/a$ states excluded by grid.

Test by measuring symmetry restoration.

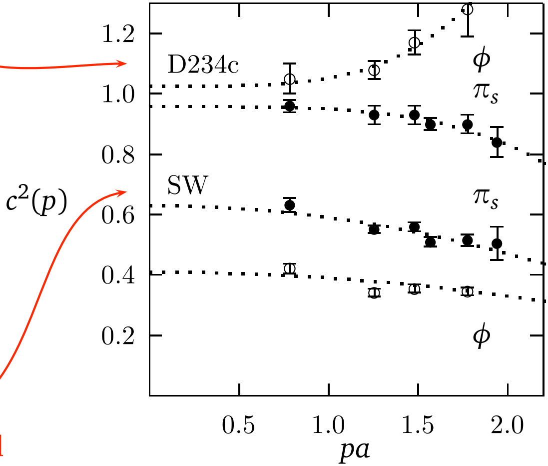
Eg) Lorentz and rotation invariance only approximate.
Test discretization by computing (for different mesons):

$$c^2(\mathbf{p}) \equiv \frac{E^2(\mathbf{p}) - m^2}{\mathbf{p}^2}.$$

Lorentz invariance implies:

$$c^2(\mathbf{p}) = 1 \qquad \forall\, \mathbf{p}.$$

Improved

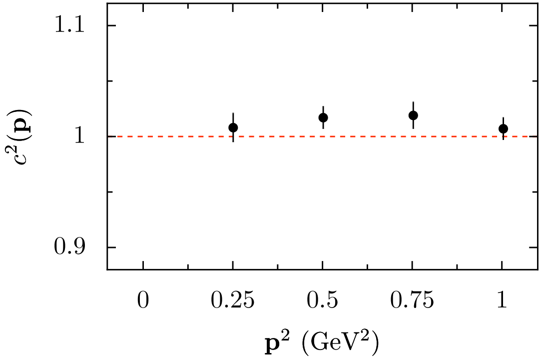


Standard

Alford et al (1997).

N.B. Much higher standards today.

Eg., c^2 for η_c, with $m_c = 0.67/a$, using HISQ action (no free parameters):



Follana et al (2007).

Quark Masses

Use perturbation theory (through $\mathcal{O}(\alpha_s^2)$) to convert bare quark masses to $\overline{\mathrm{MS}}$ masses:

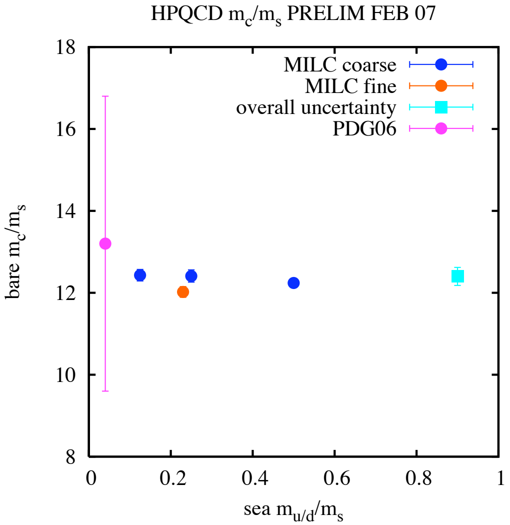
$$
\begin{aligned}
m_s^{\overline{\mathrm{MS}}}(2\,\mathrm{GeV}) &= 87\,(0)(4)(4)(0)\,\mathrm{MeV} \\
m_u^{\overline{\mathrm{MS}}} &= 1.9\,(0)(1)(1)(2)\,\mathrm{MeV} \\
m_d^{\overline{\mathrm{MS}}} &= 4.4\,(0)(2)(2)(0)\,\mathrm{MeV} \\
m_{u,d}^{\overline{\mathrm{MS}}} &= 3.2\,(0)(2)(2)(0)\,\mathrm{MeV} \\
m_s^{\overline{\mathrm{MS}}}/m_{u,d}^{\overline{\mathrm{MS}}} &= 27.4\,(1)(4)(0)(1)
\end{aligned}
$$

errors = (stat.) (extrap.) (pert. th.) (elect-mag.)

Heavy quark masses to similar precision soon.

Mason el al (HPQCD) (2005); Aubin et al (HPQCD, MILC, UKQCD) (2004)

Work in Progress: m_c/m_s



Extracting α_s

Plan

Tuned LQCD simulation ≡ real QCD.

⇒ Measure short-distance quantity Y in simulation and compare with perturbation theory,

$$Y = \sum_{n=1}^{\infty} c_n \, \alpha_V^n(d/a),$$

to extract QCD coupling constant.

- Fit multiple as to extract estimates for uncalculated c_ns:

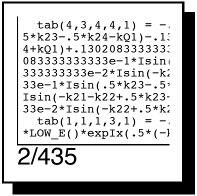
$$q^2 \frac{d\alpha_V(q)}{dq^2} = -\beta_0 \alpha_V^2 - \beta_1 \alpha_V^3 - \beta_2 \alpha_V^4 - \beta_3 \alpha_V^5 - \cdots$$

where $\alpha_0 \equiv \alpha_V(7.5\,\text{GeV})$ parameterizes $\alpha_V(q)$.

Mason et al (HPQCD), Phys. Rev. Lett. **95**, 052002 (2005).

- Perturbative calculations done with LQCD regulator and Feynman rules.
  - ◇ Results in terms of α_{latt}
  - $\Rightarrow$ Need $\alpha_{\text{latt}} \to \alpha_V$.
- Express in terms of α_V and BLM/LM scales.
  - ◇ $\alpha_V(q)$ *defined* from $V(q)$ by 3-loop expression in terms of $\alpha_{\overline{\text{MS}}}$ (and no further higher-order terms).
  - ◇ Better convergence.

N.B., vertex rule $ig\gamma_\mu$ → 435 pages of (in LQCD):



→

Method: For each short-distance quantity...

- Simulation $\Rightarrow Y_i \pm \sigma_{Y_i}$ for three different lattice spacings $\overline{a_i} \pm \sigma_{a_i}$ (1/6, 1/8, 1/11 fm).
- Lattice Pert'n theory $\Rightarrow \overline{c_n} \pm \sigma_{c_n}$ for $n \leq 3$.
- Minimize augmented χ^2

$$\chi^2 \equiv \sum_{i=1}^{3} \frac{\left(Y_i - \sum_n c_n \alpha_V^n(d/a_i)\right)^2}{\sigma_{Y_i}^2} + \sum_{n=1}^{10} \frac{(c_n - \overline{c_n})^2}{\sigma_{c_n}^2}$$
$$+ \frac{\left(\log(\alpha_0) - \overline{\log(\alpha_0)}\right)^2}{\sigma_{\log(\alpha_0)}^2} + \sum_{i=1}^{3} \frac{(a_i - \overline{a_i})^2}{\sigma_{a_i}^2},$$

  by varying $\alpha_0 \equiv \alpha_V(7.5\,\text{GeV})$, c_n, and a_i.
- Additional prior constraints:
  - $c_n = 0 \pm \sigma_c$ for $n \geq 4$, where $\sigma_c > |c_4|$ (Empirical Bayes).
  - $\alpha_0 = 0.20^{+0.20}_{-0.10}$ (that is, $\log(\alpha_0) = -1.6 \pm 0.7$).

Short-Distance Quantities

- 8 small Wilson loops,

$$W_{mn} \equiv \frac{1}{3} \langle 0 | \mathrm{Re\,Tr\,P\,e}^{-ig \oint_{nm} A \cdot dx} | 0 \rangle,$$

where the integral is over a closed $ma \times na$ rectangular path, and, e.g.,

$$\begin{aligned} \log W_{11} &= -3.068\,\alpha_V(3.33/a)\,(1 - 1.068\,\alpha_V \\ &\quad +1.69(4)\,\alpha_V^2 - 5(2)\,\alpha_V^3 - 1(6)\,\alpha_V^4 \cdots) ; \end{aligned}$$

$$\begin{aligned} \log W_{12} &= -5.551\,\alpha_V(3.00/a)\,(1 - 0.858\,\alpha_V \\ &\quad +1.72(4)\,\alpha_V^2 - 5(2)\,\alpha_V^3 - 1(6)\,\alpha_V^4 \cdots) . \end{aligned}$$

N.B. Very ultraviolet. Sensitivity to condensates and other nonperturbative effects depends strongly on loop size.

- Tadpole-improve or look at Creutz ratios to improve convergence: e.g.,

$$\log\left(\frac{W_{12}}{u_0^6}\right) = 0.949\,\alpha_V(1.82/a)\,(1+0.160(2)\,\alpha_V -0.54(8)\,\alpha_V^2-2(1)\,\alpha_V^3-0(2)\,\alpha_V^4\cdots);$$

$$\log\left(\frac{W_{13}}{W_{22}}\right) = -1.323\,\alpha_V(1.21/a)\,(1-0.39(1)\,\alpha_V +0.3(2)\,\alpha_V^2-2(1)\,\alpha_V^3+0(2)\,\alpha_V^4\cdots).$$

N.B. Much more infrared ⇒ tests different physics from W_{ij}s. Smaller c_4s, but α_Vs larger.

- Minimize reliance on lattice pert'n theory by examining renormalized quantity, $V(r) - V(a)$, where

$$V(r) = -C_F \frac{\alpha_V(0.5614/r)}{r} \left(1 + \frac{\beta_0^2}{48}\,\alpha_V^2 + \cdots\right)$$

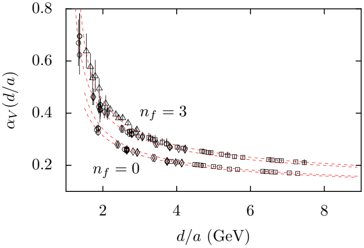
  and $C_F = 4/3$ and $\beta_0 = 11 - 2n_f/3$.

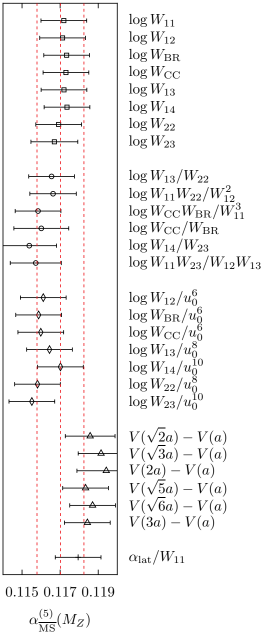
  N.B. Use lattice pert'n theory to remove small residual lattice artifacts through 2^{nd} order.

  N.B. Most infrared quantity: nonanalytic $\alpha_V^4 \log(\alpha_V)$ terms.

Results

- 28 short-distance quantities.
- $\alpha^{(5)}_{\overline{\text{MS}}}(M_Z) = 0.1170(12)$.
- PDG world average (2004) is 0.1187(20)
- Assume PDG result $\Rightarrow$ $n_f = 3.1(2)$.



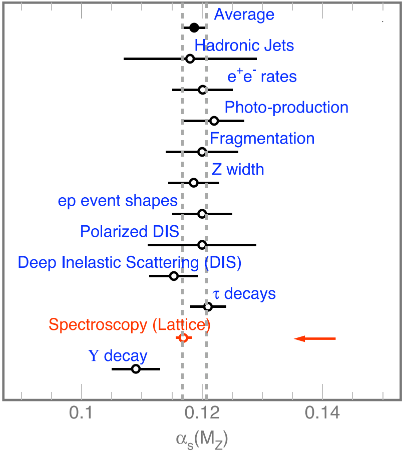


Systematic/Statistical Errors

$\alpha^{(5)}_{\overline{\mathrm{MS}}}(M_Z) = 0.1170(12)$

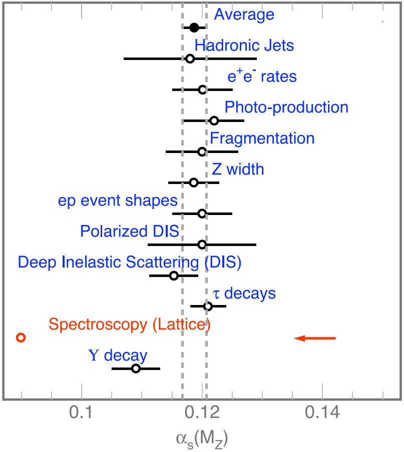
	$\log W_{11}$	$\log W_{13}/W_{22}$	$V(\sqrt{2}a) - V(a)$
a^{-1}	0.0007	0.0010	0.0010
$c_1 \dots c_3$	0.0001	0.0004	0.0004
c_n for $n \geq 4$	0.0008	0.0005	0.0004
$V \to \overline{\mathrm{MS}} \to M_Z$	0.0001	0.0001	0.0001
condensate	0.0002	0.0001	0.0001
m_u, m_d, m_s	0.0004	0.0001	0.0001
m_c, m_b	0.0002	0.0002	0.0002
simulation errors	0.0000	0.0000	0.0002
total uncertainty	0.0012	0.0012	0.0012

Context:



Mason et al, Phys. Rev. Lett. 95:052002,2005; Particle Data Group (2004)

And without light-quark vacuum polarization:



Mason et al, Phys. Rev. Lett. 95:052002,2005; Particle Data Group (2004)

Conclusions

- $\alpha^{(5)}_{\overline{\rm MS}}(M_Z) = 0.1170(12)$
  - Vacuum polarization from all three light quarks.
  - Quark and gluon actions corrected through $\mathcal{O}(a^2)$.
  - Fifteen independent determinations of a.
  - 28 different short-distance quantities.
  - Third-order perturbation theory + systematic estimates of fourth-order and beyond.
  - Agrees with world average $\Rightarrow$ nonperturbative QCD of confinement is same theory as perturbative QCD of jets; and lattice QCD has both.

- Further improvement ($< 1\%$) requires:
  - Smaller errors on a (different determinations).
  - Smaller integration errors c_1–c_3.
  - Better estimates for $c_4 \ldots$ (more lattice spacings?).

Appendix: Naive/Staggered Quarks

Doubling Problem

Simplest discretization of light quarks,

$$\mathcal{L} = \overline{\psi}(x)(\Delta \cdot \gamma + m)\psi(x)$$

$\Rightarrow$ an exact "doubling" symmetry:

$$\begin{aligned} \psi(x) \quad \rightarrow \quad \tilde{\psi}(x) &\equiv i\gamma_5\gamma_\rho\,(-1)^{x_\rho/a}\,\psi(x) \\ &= i\gamma_5\gamma_\rho\,\exp(ix_\rho\pi/a)\,\psi(x). \end{aligned}$$

$\Rightarrow$

Any low-energy mode ψ	$\equiv$	Another mode, $\tilde{\psi}$, with $p_\rho \approx \pi/a$.

max. p on lattice

General case:

$$\psi(x) \rightarrow \mathcal{B}_\zeta(x)\,\psi(x)$$

where

$$\mathcal{B}_\zeta(x) \equiv \prod_\rho \left(i\gamma_5\gamma_\rho\right)^{\zeta_\rho} \; \exp(ix\cdot\zeta\,\pi/a)$$

$\zeta = (1,0,0,0), (0,1,0,0)\ldots(1,1,0,0)\ldots$, 15 in all.

$\Rightarrow$ 1 field $\psi(x)$ creates 16 *different* but *exactly equivalent* flavors or "tastes" of quark ($p \approx \zeta\pi/a$)!

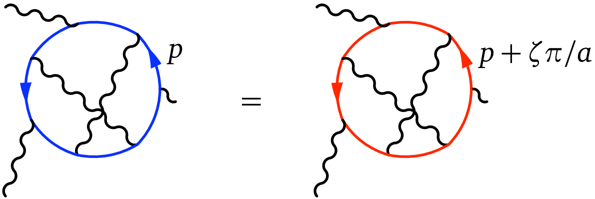
16 tastes is bad!

Two traditional options:

1. (Wilson, SW . . .) Break doubling symmtry by adding $-a\overline{\psi}(D\cdot\gamma)^2\psi/2$ to $\mathscr{L}$; destroys all chiral symmetry ⇒ small $m_{u,d}$ very difficult.

2. (Kogut-Susskind . . .) Live with the 16 tastes by inserting factors of 1/16 in strategic places. Preserves a chiral symmetry ⇒ small masses relatively efficient.

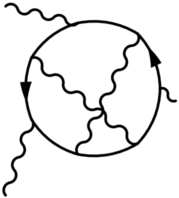
E.g.) Low-p ($\ll \pi/a$) Physics:



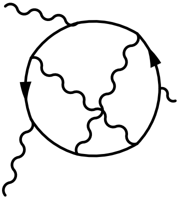
- Exact and nonperturbative.

  $\Rightarrow$ 16 identical copies of same physics.

Remove redundant copies:



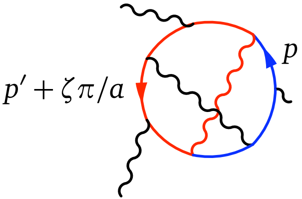
$$\rightarrow \frac{1}{16}$$



$\Rightarrow$ in path integral:

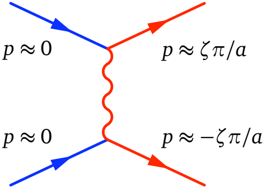
$$\det(\Delta \cdot \gamma + m) \rightarrow [\det(\Delta \cdot \gamma + m)]^{1/16}$$

But counting spoiled by:



$\Rightarrow$ Taste changes along quark line.

Bad News: Taste-changing strong interactions —



Quarks on-shell, but different taste.

Good News: Gluon carries largest lattice momentum, $\zeta\pi/a$.

- $\Rightarrow$ Highly virtual and perturbative.
- $\Rightarrow$ Taste-exchange interaction $\equiv a^2(\overline{\psi}\gamma\psi)^2$.
- $\Rightarrow$ Taste-exchange effects vanish as $a \to 0$.
  - Chiral + lattice QCD weak-coupling perturbation theory $\Rightarrow$ still true with 1/16 root.
  - Minimum (positive) quark mass $\propto a^2$.
- $\Rightarrow$ Remove taste-exchange effects using improved discritization.
  - Cancel tree-level taste-exchange $\Rightarrow$ ASQTAD.
  - Cancel one-loop taste-exchange $\Rightarrow$ HISQ.

See: Lepage and Toussaint (1997); Lepage (1998); MILC (1999); Follana et al (HPQCD 2007). Review by Sharpe (2006).

Staggered Quarks

$$\Omega(x) \equiv \prod_{\mu} (\gamma_\mu)^{x_\mu/a}$$

$$\Rightarrow \qquad S_F(x,y;A_\mu) = g(x,y;A_\mu)\,\Omega(x)\,\Omega^\dagger(y)$$

Propagator.

Dirac scalar for any gluon field A_μ.

Dirac structure is completely independent of A_μ!

$\Rightarrow$ 50–1000 times faster than other alternatives!