Status of the Strange Quark Mass Determination

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$\overline{m}_q(2\ \text{GeV}),\ q = u, d, s$

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$\sim 25\%$ accuracy ,

compared with $\sim 10\%$ for $\overline{m}_c(\overline{m}_c)$ and $\sim 2\%$ for $\overline{m}_b(\overline{m}_b)$

- ChPT yields the relations:

$$R = \frac{m_s}{\hat{m}} = 24.4 \pm 1.5, \quad \mathcal{Q}^2 = \frac{m_s^2 - \hat{m}^2}{m_d^2 - m_u^2} = (22.7 \pm 0.8)^2$$

[Leutwyler, 1996]

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$\hat{m} = \frac{1}{2}(m_u + m_d)$

determine m_s, obtain $m_{u,d}$ "for free ":

$$m_d = \frac{m_s}{R}\left(1 + \frac{R-1}{4Q^2}\right)$$
$$m_u = \frac{m_s}{R}\left(1 - \frac{R-1}{4Q^2}\right)$$

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  - ∗ inclusive $\tau \longrightarrow s\bar{u}\nu_\tau$ decays
  - ∗ Finite-energy sum rules (FESR)

Lattice QCD results ($\geq$ 2005)

n_f	$\overline{m}_s$(2GeV)[MeV]	Ref.
2	97 ± 22	[ALPHA Collab.] Della Morte et al Nucl.Phys. B (2005)
2	$101 \pm 8^{+25}_{-0}$	[SPQ$_{CD}$R Collab.] Becirevic et al. Proc.Lat2005
2	$115 \pm 2 \pm 3 \pm 6$	[QCDSF-UKQCD Collab.] M. Göckeler et al. Proc.Lat2006
3	$87 \pm 4 \pm 4$	[HPQCD Collab.] Mason et al. PRD (2006)
3	$91.8 \pm 3.9^{+6.8}_{-4.1}$	[CP-PACS & JLQCD Collab.], Ishikawa et al., Proc.Lat2006
3	$90 \pm 5 \pm 4$	[MILC Collab.] Bernard et al.Proc.Lat2006
2,3	95 ± 20	lattice average, C. T. Sachraida, A. V. Manohar PDG review on m_q (2006)

Employing quark-current correlators

$$\Pi(q^2) = i\int d^4x\; e^{iq\cdot x}\langle 0|T\left\{j^{(s)}(x)j^{(s)\dagger}(0)\right\}|0\rangle$$

$$j^{(s)} = (\bar{s}q)_{V,A,S,P,\ldots}$$

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$*$ OPE at $Q^2 = -q^2 \gg \Lambda^2_{QCD}$

(PQCD $\oplus$ condensates)

$$\Pi(Q^2) = \sum_{d=1,2,3,\ldots} \frac{C_d(\alpha_s, m_s, \ldots)}{(Q^2)^d} \quad .$$

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$$\Pi(Q^2) = \sum_{d=1,2,3,\dots} \frac{C_d(\alpha_s, m_s, \dots)}{(Q^2)^d} = \frac{1}{\pi} \int_0^\infty ds \frac{\mathrm{Im}\Pi(s)}{(s+Q^2)} \; (+subtr.)$$

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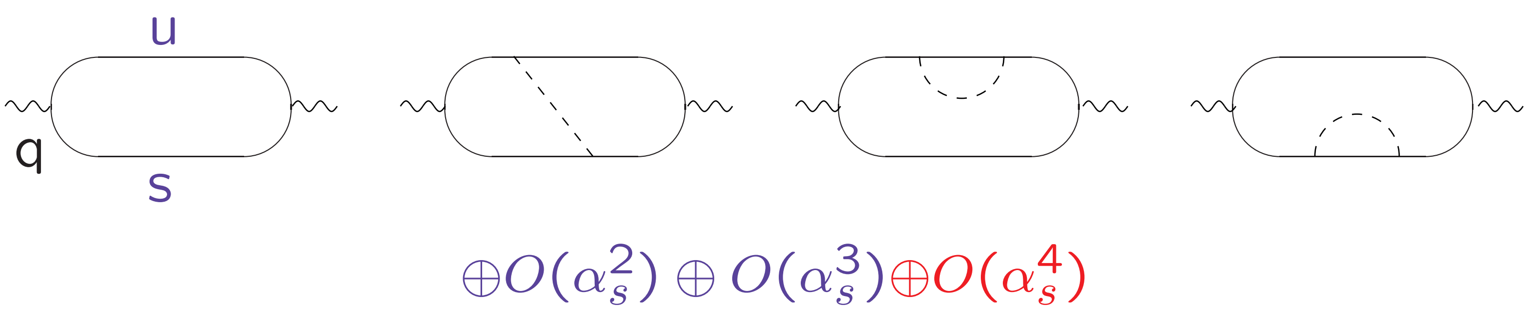
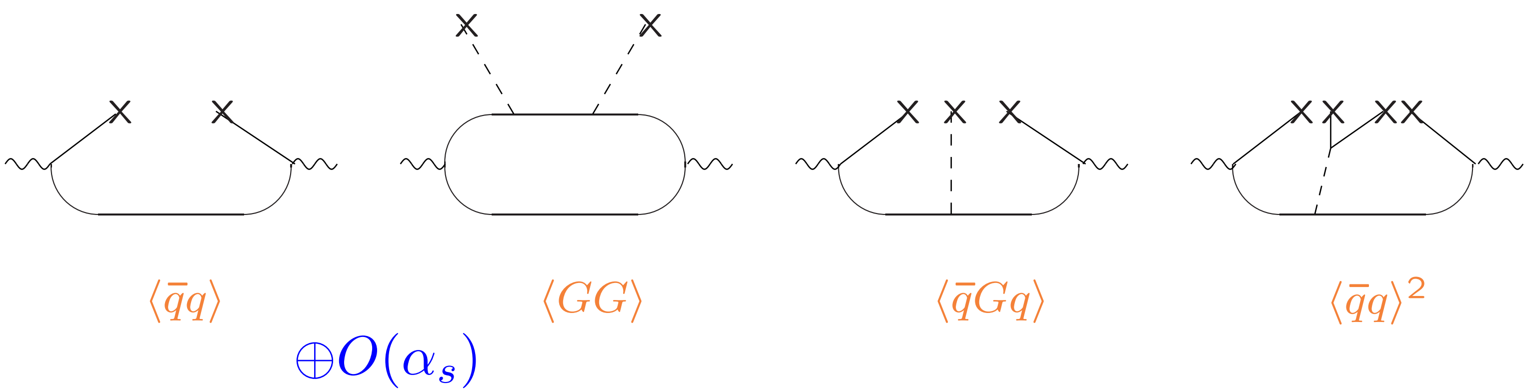
* unitarity:

$$\frac{1}{\pi}\mathrm{Im}\Pi(s) = \sum_{h_s} \langle 0 | j^{(s)} | h_s \rangle \langle h_s | j^{(s)} | 0 \rangle \Leftarrow \text{hadronic observables}$$

e.g., $j_P^{(s)} = \bar{s}\gamma_5 q$, $\quad h_s = K, K2\pi, K^*\pi, \ldots$

OPE: perturbative and condensate diagrams





.

Current status of OPE

- Correlators with vector and axial currents (employed in τ-decays): calculated to order α_s^3 [Baikov, Chetyrkin, Kühn, PRL(2005)]

- Correlators with scalar (pseudoscalar) currents:

$$j^{(s)}_{S(P)} = \partial^\mu \bar{s}\gamma_\mu(\gamma_5)q = (m_s \,(\mp)\, m_q)\bar{s}(\gamma_5)q, \quad (q = u, d)$$

$$\Pi(Q^2) \; = (m_s \mp m_q)^2 \widetilde{\Pi}(Q^2)$$

calculated to order α_s^4 [Baikov, Chetyrkin, Kühn, PRL(2006)]

OPE for the pseudoscalar correlator

expansion in $1/(Q)^{d+2}$, $d = 0, 2, 4, 6$

$$[\Pi^{(P)''}(Q^2)]_{OPE} = \frac{3(m_s + m_u)^2}{8\pi^2 Q^2} \left\{ 1 + \sum_{i=1}^{4} C_{0,i} \left(\frac{\alpha_s}{\pi}\right)^i \right.$$

$$\left. -2\frac{m_s^2}{Q^2} \left(1 + \sum_{i=1,2} C_{2,i} \left(\frac{\alpha_s}{\pi}\right)^i \right) + \frac{\{d=4\}}{Q^4} + \frac{\{d=6\}}{Q^6} \right\}$$

$\{d = 4\} \sim \{m_s \langle \bar{q} q \rangle, \langle G^2 \rangle, O(m_s^4)\ \}\ (1 \oplus O(\alpha_s))$

$\{d = 6\} \sim m_s \langle \bar{q} G q \rangle, \langle \bar{q} q \rangle^2$

Coefficients multiplying $(\alpha_s/\pi)^n$ in $d = 0$ part: $(l_Q = \log Q^2/\mu^2)$

$$C_{0,1} = \frac{11}{3} - 2\, l_Q\,, \quad C_{0,2} = \frac{5071}{144} - \frac{35}{2}\,\zeta_3 - \frac{139}{6}\, l_Q + \frac{17}{4}\, l_Q^2,$$

$$C_{0,3} = \frac{1995097}{5184} - \frac{\pi^4}{36} - \frac{65869}{216}\,\zeta_3 + \frac{715}{12}\,\zeta_5 - \frac{2720}{9}\, l_Q + \frac{475}{4}\,\zeta_3\, l_Q + \frac{695}{8}\, l_Q^2 - \frac{221}{24} l_Q^3,$$

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new:

$$C_{0,4} = \frac{2361295759}{497664} - \frac{2915}{10368}\pi^4 - \frac{25214831}{5184}\,\zeta_3 + \frac{192155}{216}\,\zeta_3^2 + \frac{59875}{108}\,\zeta_5 - \frac{625}{48}\,\zeta_6$$
$$- \frac{52255}{256}\,\zeta_7 + l_Q\left[-\frac{43647875}{10368} + \frac{1}{18}\pi^4 + \frac{864685}{288}\,\zeta_3 - \frac{24025}{48}\,\zeta_5\right]$$
$$+ l_Q^2\left[\frac{1778273}{1152} - \frac{16785}{32}\,\zeta_3\right] + l_Q^3\left[-\frac{79333}{288}\right] + l_Q^4\left[\frac{7735}{384}\right],$$

Hierarchy in α_s and d

Relative contributions to $[\Pi^{(P)''}(M^2)]_{OPE}$ (after Borel transf. $Q^2 \to M^2$)

$$r_n^{(d)}(M^2) = \frac{\{\Pi^{(P)''}(M^2)\}^{(d)}_{O(\alpha_s^n)}}{\Pi^{(P)''}(M^2)}$$

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$r_n^{(d=0,2)}(2.5\ \mathrm{GeV}^2) = 52.4\%,\ 28.3\%,\ 14.4\%,\ 4.0\%,\ -0.3\%$, $n = 0, 1, 2, 3, 4,$

$r^{(d=4,6)}(2.5\ \mathrm{GeV}^2) = 1.2\%$.

Bounds for m_s

- positivity of the spectral function e.g., in the pseudoscalar channel:

$$\frac{1}{\pi}\mathrm{Im}\Pi^{(P)}(s) \; = f_K^2 m_K^4 \delta(m_K^2 - s) + \{\geq 0\}$$

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- double-subtracted dispersion relation $\Pi^{(P)''}(Q^2)$ yields the linear bound:

$$[m_s(Q^2) + m_q(Q^2)]^2 \geq \frac{32\pi^2}{3Q^4} \frac{f_K^2 m_K^4}{(1 + m_K^2/Q^2)^3} \left(\frac{1}{1 + O(\alpha_s) + ...} \right)$$

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- power moments: $\mathcal{F}_n(Q^2) = c_n (Q^2)^{n+1}\left(\frac{d}{dQ^2}\right)^{n+2}\Pi''(Q^2)$

$\Rightarrow$ the quadratic bound, combining $\mathcal{F}_{0,1,2}$,

. [Lellouch, de Rafael, Taron (1997)]

- recent $O(\alpha_s^4)$ improvement [Baikov, Chetyrkin, Kühn, PRL(2006)]
  - * consistent α_s-expansion of the ratio of moments
  - * good convergence of perturb. series

Bound	$[\overline{m}_s + \overline{m}_u](2\text{GeV})$	precision
linear	> 72 MeV	$O(\alpha_s^4)$
quadratic	> 77 MeV	$O(\alpha_s^4)$

QCD sum rule (pseudoscalar, Borel)

Dispersion relation for $\Pi^{(P)}(q^2)$ at large Q^2 (doubly differentiated):

$$[\Pi^{(P)''}(q^2)]_{OPE} = 2 \int_0^\infty ds \frac{\rho^{(P)}(s)}{(s-q^2)^3} \, .$$

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$$\rho^{(P)}(s) = \sum_K \langle 0|j_P^{(s)}|K(q)\rangle\langle K(q)|j_P^{(s)}|0\rangle$$

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Borel transformation:

$$M^4[\Pi^{(P)''}(M^2)]_{OPE} = \int\limits_0^{s_0} ds\, e^{-s/M^2} \rho^{(P)}_{hadr}(s) + \int\limits_{s_0}^\infty ds\, e^{-s/M^2} \rho^{(P)}_{OPE}(s) \,.$$

The hadronic part

- $K, K2\pi, K^*\pi, \rho K, \ldots \to$ three-resonance ansatz $\{K, K_1(1460), K_2(1830))\}$

m_{K_1}=1460 MeV, Γ_{K_1} = 260 MeV; m_{K_2}=1830 MeV, Γ_{K_2}=250 MeV [PDG]

$$\rho^{(P)}_{hadr}(s) = f_K^2 m_K^4 \delta(m_K^2 - s) + \sum_{i=1,2} f_{K_i}^2 m_{K_i}^4 \frac{1}{\pi}\left(\frac{\Gamma_{K_i} m_{K_i}}{(s - m_K^2)^2 + (\Gamma_{K_i} m_{K_i})^2}\right)$$

- decay constants: $\langle 0|j_5^{(s)}|K(q)\rangle = f_K m_K^2$, $f_K = 159.8$ MeV ,

$f_{K_1} = \sqrt{2}(22.9 \pm 2.4)$ MeV, $f_{K_2} = \sqrt{2}(14.5 \pm 1.5)$ MeV, $s_0 = 4.5 \pm 0.5$ GeV^2

fitted from pseudoscalar sum rules and FESR with $O(\alpha_s^3)$ accuracy

[Kambor, Maltman , PRD 2002] (small f_{K_i} consistent with PCAC/ChPT)

$O(\alpha_s^4)$ improvement of the pseudoscalar sum rule

. [Chetyrkin, A.K. , EJP (2006)]

- inputs: $\alpha_s(m_Z)$, $(\alpha_s(m_\tau))$, condensates

$\overline{m}_u(2\ \mathrm{GeV}) = (1.5 - 5.0)$ MeV,
$\overline{m}_d(2\ \mathrm{GeV}) = (5.0 - 9.0)$ MeV (non-lattice) intervals from [PDG])

- Borel mass interval: $2 < M^2 < 3\ \mathrm{GeV}^2$,

(to avoid instantons and to minimize the contribution of excited states)

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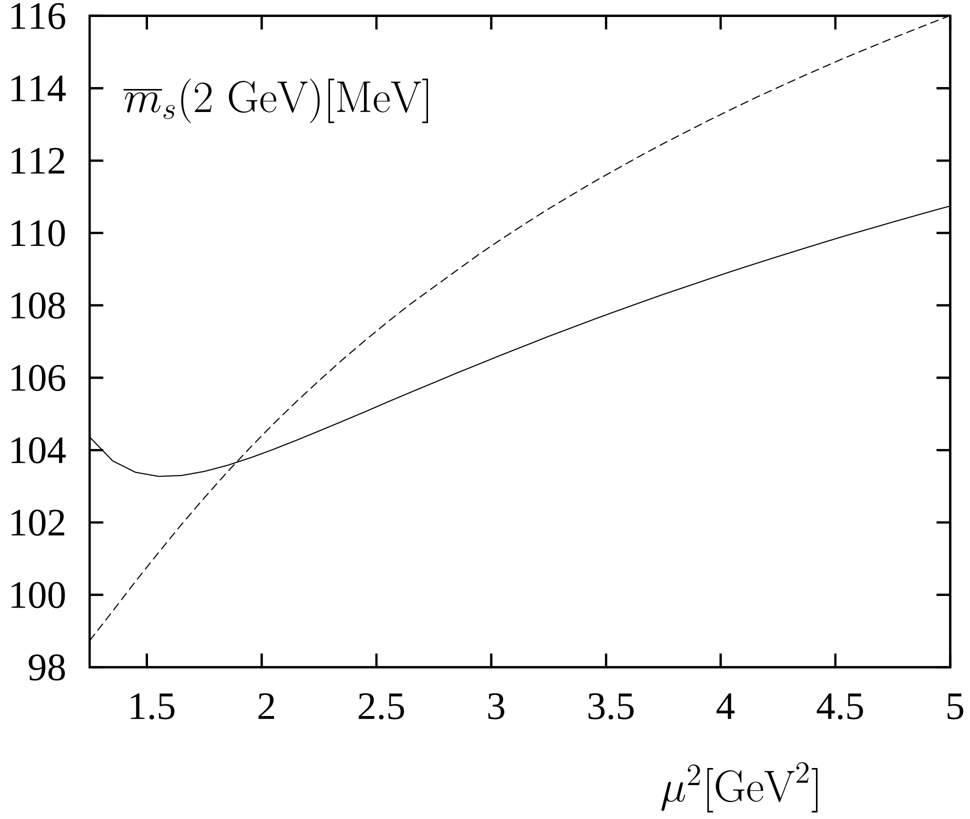
- the result: $$\overline{m}_s(2\ \text{GeV}) = \left(105 \pm 6\Big|_{OPE} \pm 7\Big|_{hadr}\right)\text{MeV}\,,$$

$\simeq 2$ MeV increase of the central value, if the $O(\alpha_s^4)$ terms are removed.

The scale dependence



solid $O(\alpha_s^4)$, dashed $O(\alpha_s^3)$

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$(K_1 \to K^*\pi, K\rho, K2\pi, ...)$,

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($K_1 \to K^*\pi, K\rho, K2\pi, ...$),
but: $BR \sim 10^{-5}$ -chirally and Cabibbo suppressed

- excited kaons in semileptonic charmed meson decays,
e.g., $D \to K_1(1460) l \nu_l$,
no chiral/Cabibbo suppression, but: no access to f_{K_1}

- kaon resonances in $B \to J/\psi K \pi\pi$ etc. decays, $BR \sim 10^{-4} - 10^{-3}$

other QCD sum rules

- use of the scalar correlator

* the same OPE as in the pseudoscalar case $(m_s + m_u)^2 \longrightarrow (m_s - m_u)^2$

* a different pattern of the hadronic spectral function, $J^P = O^+$ $K\pi$ -state

a reliable knowledge of the $K \longrightarrow \pi$ scalar form factor is needed

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- FESR (scalar, pseudoscalar) [Maltman, Kambor,'02]

vector FESR [Eidemüller, Jamin,'03]

m_s from $\tau \to s\bar{u}\nu_\tau$ decays

* based on vector/axial-vector correlators

* $\oint \Pi(Q^2) dQ^2$ related to $\int_0^{m_\tau^2} ds Im\Pi(s)$, where $\mathrm{Im}\Pi(s) \sim R_\tau(s)$

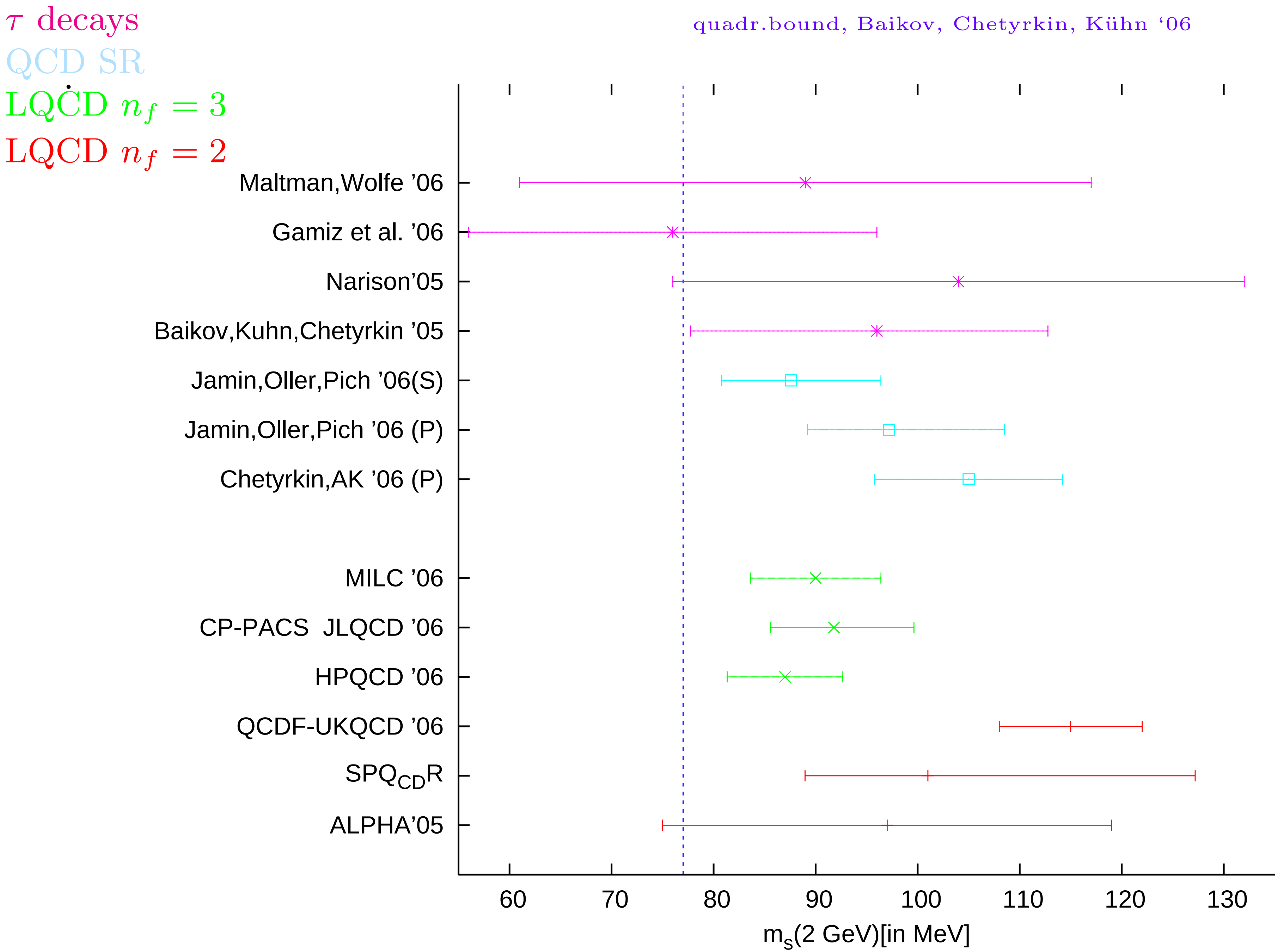
* the art of combining the moments of spectral densities [K. Maltman et al.]

* less sensitive to m_s, serves primarily for a precise extraction of $|V_{us}|$

(see the next talk by Matthias Jamin)

Non-lattice determinations (≥ 2005)

Method	$\overline{m}_s(2\text{ GeV})$ in MeV	Ref.
Hadronic τ decays	96^{+5+16}_{-3-18} 104 ± 28 76 ± 20 89 ± 28	Baikov, Chetyrkin, Kühn, PRL (2005) S. Narison, hep-ph/0510108 Gamiz et al, hep-ph/0612154 Maltman,Wolfe, hep-ph/0701037
Pseudoscalar Borel SR	$105 \pm 6 \pm 7$ $97.2^{+11.3}_{-8.0}$	A.K., Chetyrkin, hep-ph/0612295 Jamin, Oller, Pich hep-ph/0605095
Scalar Borel SR	$87.6^{+8.8}_{-6.8}$	Jamin, Oller, Pich hep-ph/0605095
nonlattice average	103 ± 20	C. T. Sachraida, A. V. Manohar PDG review on m_q (2006) $O(\alpha_s^4)$ not included


τ decays
QCD SR
LQCD $n_f = 3$
LQCD $n_f = 2$
quadr.bound, Baikov, Chetyrkin, Kühn '06
Maltman,Wolfe '06
Gamiz et al. '06
Narison'05
Baikov,Kuhn,Chetyrkin '05
Jamin,Oller,Pich '06(S)
Jamin,Oller,Pich '06 (P)
Chetyrkin,AK '06 (P)
MILC '06
CP-PACS JLQCD '06
HPQCD '06
QCDF-UKQCD '06
SPQ$_{CD}$R
ALPHA'05
60
70
80
90
100
110
120
130
m_s(2 GeV)[in MeV]

Summary

- $O(\alpha_s^4)$ term in the correlators of (pseudo)scalar $\Delta S = 1$ quark currents: good convergence of OPE in both α_s and d

- the bound on m_s improved by including $O(\alpha_s^4)$

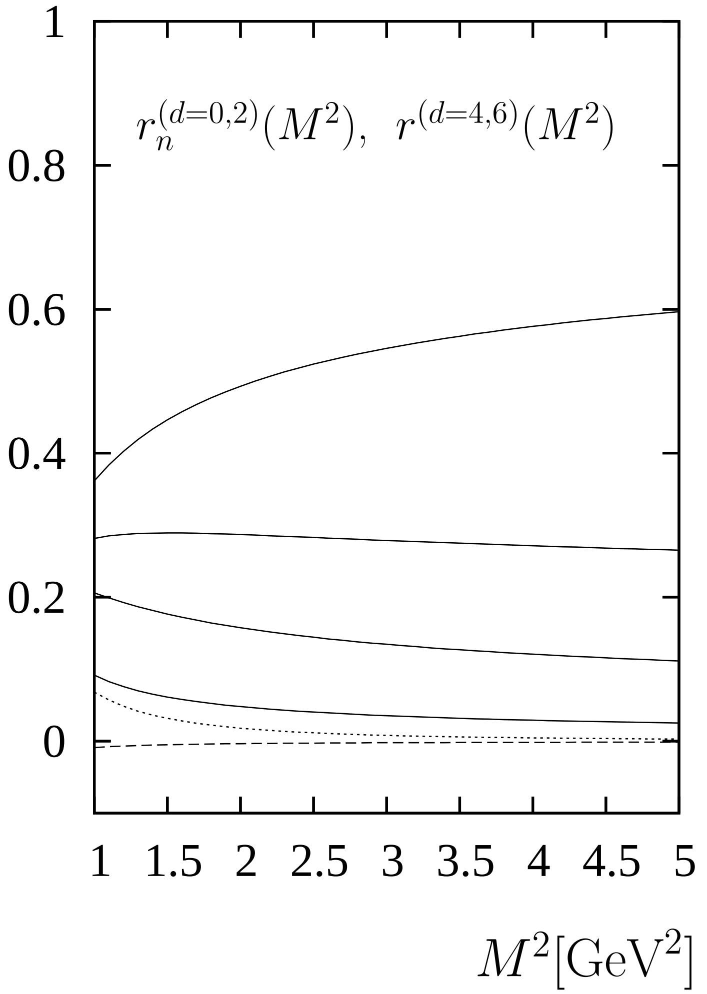
- QCD sum rules with $O(\alpha_s^4)$ correction are less scale-dependent than in $O(\alpha_s^3)$

- hadronic input in the sum rules can be refined with more experimental data on kaon resonances, e.g., from exclusive τ , D and B decays

- lattice vs "continuum" determinations - not only a general agreement: retaining only $n_f = 3$ lattice and $O(\alpha_s^4)$ QCDSR, (PDG06) interval $m_s(2\text{GeV}) = 70 - 120$ MeV $\Rightarrow$ $m_s(2\text{GeV}) = 82 - 115$ MeV.

Backup Slides

•Relative contributions to OPE: dependence on Borel mass



solid (from up to down): $r_n^{(d=0,2)}$ with $n = 0, 1, 2, 3$, dashed: $r_4^{(d=0,2)}$, dotted: $r^{(d=4,6)}$.

Results for $\overline{m}_s(2\ \mathrm{GeV})$, as end of 2005

Pseudoscalar Borel sum rule	$105 \pm 6 \pm 7$	This work
	100 ± 6	[Maltman, Kambor '00]
Pseudoscalar FESR	100 ± 12	[Maltman, Kambor '00]
Scalar Borel sum rule	99 ± 16	[Jamin,Oller,Pich '02]
Vector FESR	139 ± 31	[Eidemuller,Jamin,Schwab '03]
Hadronic τ decays	81 ± 22	[Gamiz et al,'05]
	96^{+5+16}_{-3-18}	[Baikov, Chetyrkin, Kühn,'05]
	104 ± 28	[Narison'05]
τ decays $\oplus$ sum rules	99 ± 28	[Narison'05]
Lattice QCD ($n_f = 2$)	97 ± 22	[ALPHA Collab.'05]
	$111 \pm 6 \pm 4 \pm 6$	[QCDSF-UKQCD Collab. '06]
	$101 \pm 8^{+25}_{-0}$	[SPQcdR Collab.'05]
Lattice QCD ($n_f = 3$)	$76 \pm 3 \pm 7$	[HPQCD-MILC-UKQCD Collab.
	86.7 ± 5.9	[CP-PACS-JLQCD Colllab. '05]
	$87 \pm 4 \pm 4$	[HPQCD '05]
PDG average	80 -130	[Particle Data Group]