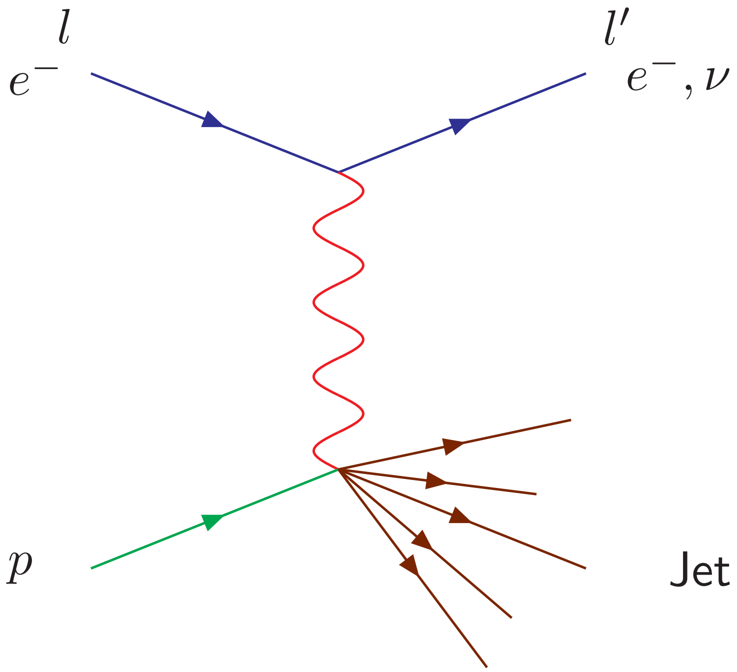
Λ_{QCD} and $\alpha_s(M_Z^2)$ from DIS Structure Functions

Johannes Blümlein
DESY

- Introduction and Method
- QCD Analysis of Unpolarized Structure Functions
- QCD Analysis of Polarized Structure Functions
- Λ_{QCD} and $\alpha_s(M_Z^2)$
- What would we like to know ?

Wilhelmus Ludovicus van Neerven
1947 – 2007

Deeply Inelastic Scattering



$$\textbf{space} - \textbf{like process} \quad : \quad q^2 = (l - l')^2 = -Q^2 < 0 \qquad W^2 = (p+q)^2 \geq M_p^2$$

$$x = \frac{Q^2}{2p.q}, \qquad y = \frac{p.q}{p.l} \qquad 0 \leq x, y \leq 1$$

DIS: Microscopy of the Nucleon

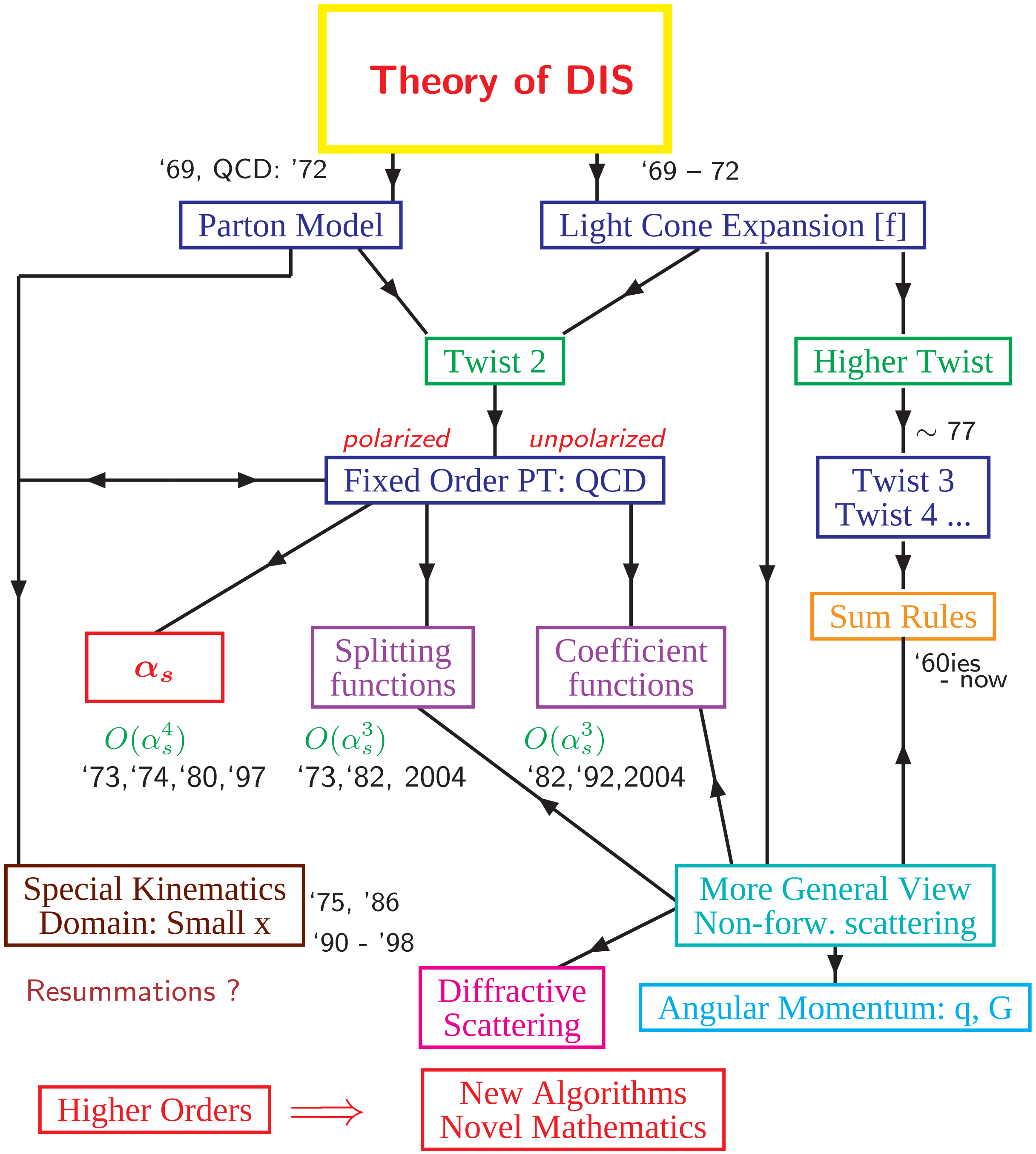
- determination of all quark densities and the gluon distribution
- determination of all polarized parton densities

DIS: Fundamental Tests of QCD

- precision measurement of Λ_{QCD} and $\alpha_s(M_Z^2)$
- Thorough verification of the prediction of the light cone expansion: to higher twist
- Test of linear and non-linear resummations

Challenges for Theory: perturbative and non-perturbative

- higher order precision calculations and data analysis
- Lattice gauge theory results for Λ_{QCD} and hadronic ME


Theory of DIS
'69, QCD: '72
'69 – 72
Parton Model
Light Cone Expansion [f]
Twist 2
Higher Twist
∼ 77
polarized
unpolarized
Fixed Order PT: QCD
Twist 3
Twist 4 ...
Sum Rules
'60ies - now
α_s
Splitting functions
Coefficient functions
$O(\alpha_s^4)$
'73,'74,'80,'97
$O(\alpha_s^3)$
'73,'82, 2004
$O(\alpha_s^3)$
'82,'92,2004
Special Kinematics Domain: Small x
'75, '86
'90 - '98
More General View Non-forw. scattering
Resummations ?
Diffractive Scattering
Angular Momentum: q, G
Higher Orders $\Longrightarrow$
New Algorithms Novel Mathematics

Highest order corrections of HO QCD in DIS

- Running α_s: $O(\alpha_s^4)$ Larin, van Ritbergen, Vermaseren 1997
- Unpol. anomalous dimensions and Wilson coefficients: $O(\alpha_s^3)$ Moch, Vermaseren, Vogt 2004/05
- Unpol. NS anomalous dimension 2nd Moment: $O(\alpha_s^4)$ Baikov, Chetyrkin 2006
- Pol. anomalous dimension: $O(\alpha_s^2)$; $\Delta P_{NS}^{qq}, \Delta P_{qG}$: $O(\alpha_s^3)$ Mertig, van Neerven, 1995; Vogelsang 1995; Moch, Vermaseren, Vogt
- Pol. Wilson coefficients: $O(\alpha_s^2)$; $\Delta C_{NS}^{qq}, \Delta C_{qG}$: van Neerven, Zijlstra 1994 $O(\alpha_s^3)$ to come
- Transversity: $O(\alpha_s^2)$, some moments anom. dim.: $O(\alpha_s^3)$, Hayashigaki, Kanazawa, Koike; Kumano, Miyama; Vogelsang; 1997; Gracey 2006
- Unpol. Heavy Flavor Wilson Coefficients: $O(\alpha_s^2)$ Laenen, van Neerven, Riemersma, Smith, 1993
  Fast Mellin Space code: Blümlein & Alekhin, 2003
- Pol. Heavy Flavor Wilson Coefficients: $O(\alpha_s^1)$, Watson 1982
- $Q^2 \gg m^2$ Pol. Heavy Flavor Wilson Coefficient : $O(\alpha_s^2)$ van Neerven, Smith et al. 1996, Blümlein & Klein 2007
- $Q^2 \gg m^2$ Unpol. Heavy Flavor Wilson Coefficient F_L: $O(\alpha_s^3)$
  Blümlein, De Freitas, van Neerven, S. Klein 2005

DIS Structure Functions @ Twist 2

$$F_j(x,Q^2) = \hat{f}_i(x,\mu^2) \otimes \sigma_j^i \left(\alpha_s, \frac{Q^2}{\mu^2}, x\right)$$

$$\uparrow \text{bare pdf} \qquad \uparrow \text{sub} - \text{system cross} - \text{sect.}$$

$$= \underbrace{\hat{f}_i(x,\mu^2) \otimes \Gamma_k^i \left(\alpha_s(R^2), \frac{M^2}{\mu^2}, \frac{M^2}{R^2}\right)}_{\text{finite pdf} \equiv f_k}$$

$$\otimes \underbrace{C_j^k \left(\alpha_s(R^2), \frac{Q^2}{\mu^2}, \frac{M^2}{R^2}, x\right)}_{\text{finite Wilson coefficient}}$$

Move to Mellin space :

$$F_j(N) = \int_0^1 dx x^{N-1} F_j(x)$$

Diagonalization of the convolutions $\otimes$ into ordinary products.

Evolution Equations

$$\left[M\frac{\partial}{\partial M}+\beta(g)\frac{\partial}{\partial g}-2\gamma_\psi(g)\right]F_i(N)=0$$

$$\left[M\frac{\partial}{\partial M}+\beta(g)\frac{\partial}{\partial g}+\gamma_\kappa^N(g)-2\gamma_\psi(g)\right]f_k(N)=0$$

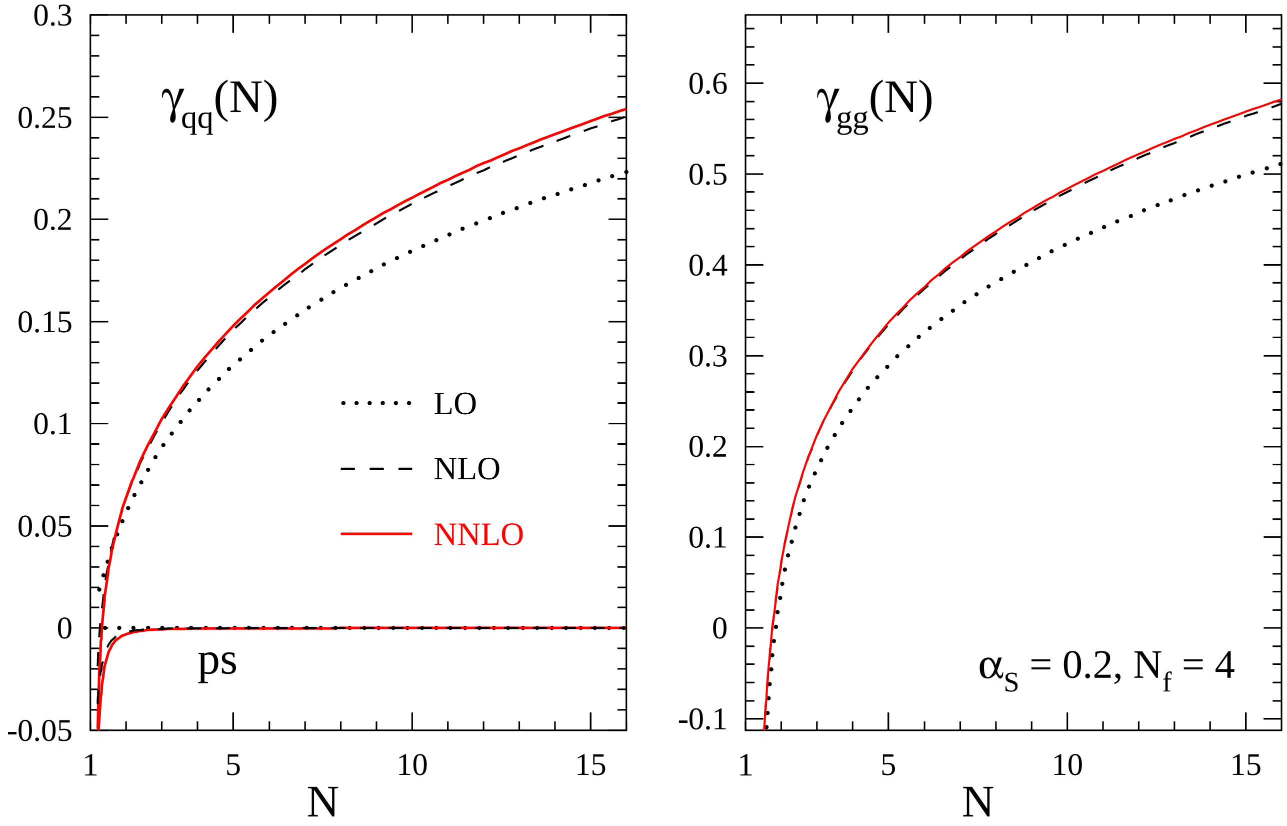
$$\left[M\frac{\partial}{\partial M}+\beta(g)\frac{\partial}{\partial g}-\gamma_\kappa^N(g)\right]C_j^k(N)=0$$

CALLAN–SYMNANZIK equations for mass factorization
$\equiv$ ALTARELLI–PARISI evolution equations
x-space :

$$\frac{d}{d\log(\mu^2)}\begin{pmatrix} q^+(x,Q^2)\\ G(x,Q^2)\end{pmatrix}=\frac{\alpha_s}{2\pi}\boldsymbol{P}(x,\alpha_s)\otimes\begin{pmatrix} q^+(x,Q^2)\\ G(x,Q^2)\end{pmatrix}$$

$$\boldsymbol{P}(x,\alpha_s)=\boldsymbol{P}^{(0)}(x)+\frac{\alpha_s}{2\pi}\boldsymbol{P}^{(1)}(x)+\left(\frac{\alpha_s}{2\pi}\right)^2\boldsymbol{P}^{(2)}(x)+\dots$$

Anomalous Dimensions and Wilson Coefficients



Vermaseren, Moch, Vogt 2004

The Basic Functions of massless QCD to w=5: $\equiv$ 3 Loops

Representative : $S_1(N) = \psi(N+1) + \gamma_E$ and its derivatives.

Weight w=3 :

$$F_1(N) = \mathbf{M}\left[\frac{\ln(1+x)}{1+x}\right](N)$$

$$F_2(N) = \mathbf{M}\left[\frac{\mathrm{Li}_2(x)}{1+x}\right](N), \qquad F_3(N) = \mathbf{M}\left[\left(\frac{\mathrm{Li}_2(x)}{1-x}\right)_+\right](N)$$

Yndurain et al., 1981: $F_2(N)$

Weight w=4 :

$$F_4(N) = \mathbf{M}\left[\frac{S_{1,2}(x)}{1+x}\right](N), \qquad F_5(N) := \mathbf{M}\left[\left(\frac{S_{1,2}(x)}{1-x}\right)_+\right](N)$$

$F_3(N) - F_5(N)$: J.B., S. Moch, 2003; J.B., V. Ravindran ,2004

Weight w=5 :

$$F_{6,7}(N) = \mathbf{M}\left[\left(\frac{\mathrm{Li}_4(x)}{1 \pm x}\right)_{(+)}\right](N), \qquad F_8(N) = \mathbf{M}\left[\frac{S_{1,3}(x)}{1+x}\right](N),$$

$$F_{9,10}(N) = \mathbf{M}\left[\left(\frac{S_{2,2}(x)}{1 \pm x}\right)_{(+)}\right](N), \qquad F_{11}(N) = \mathbf{M}\left[\frac{\mathrm{Li}_2^2(x)}{1+x}\right](N),$$

$$F_{12,13}(N) := \mathbf{M}\left[\left(\frac{S_{2,2}(-x) - \mathrm{Li}_2^2(-x)/2}{1 \pm x}\right)_{(+)}\right](N)$$

$F_6(N) - F_{13}(N)$: J.B., S. Moch, 2004.

Massless QCD to 3 Loops depends on 14 Functions.

$\Rightarrow$ Representation for 3 Loop Wilson Coefficients under way.

Complex Analysis of these Functions

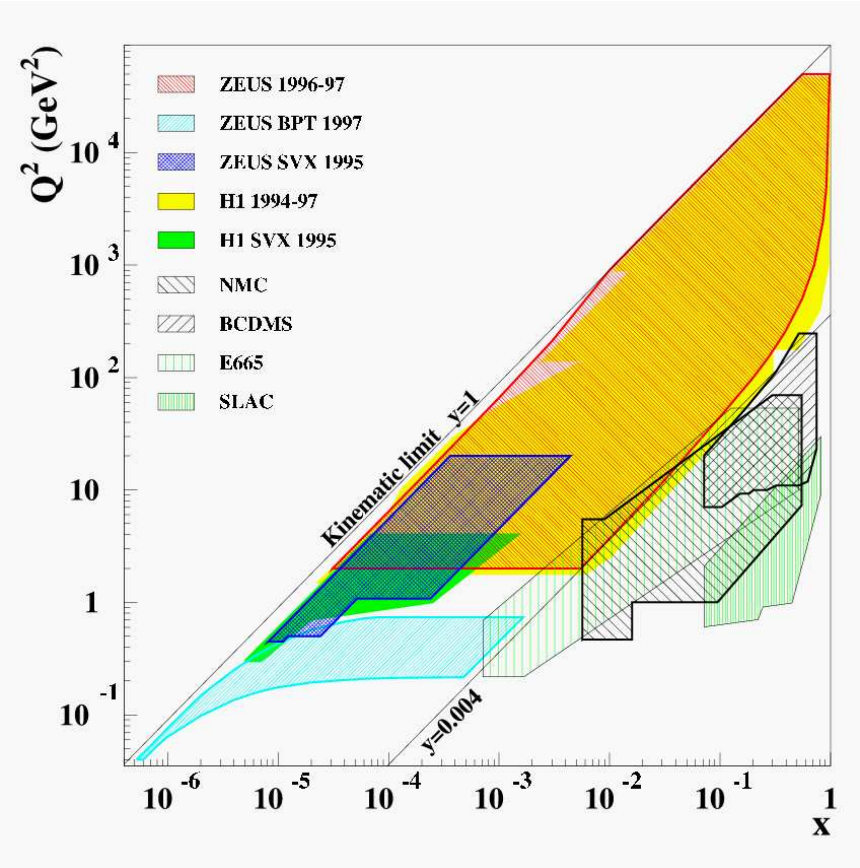
- Construct exact analytic continuations to complex N
- The functions are meromorphic
(up to soft corrections, which have a simple structure)
- Asymptotic Representation
- Recursion $z+1 \rightarrow z$
- Solve the Evolution Equations fully analytically and form an analytic expression for the Structure functions in Mellin Space at all Q^2
- Include the heavy flavor Wilson coefficients in Mellin Space
- Perform a single fast, numerical Mellin inversion
(at high precision)

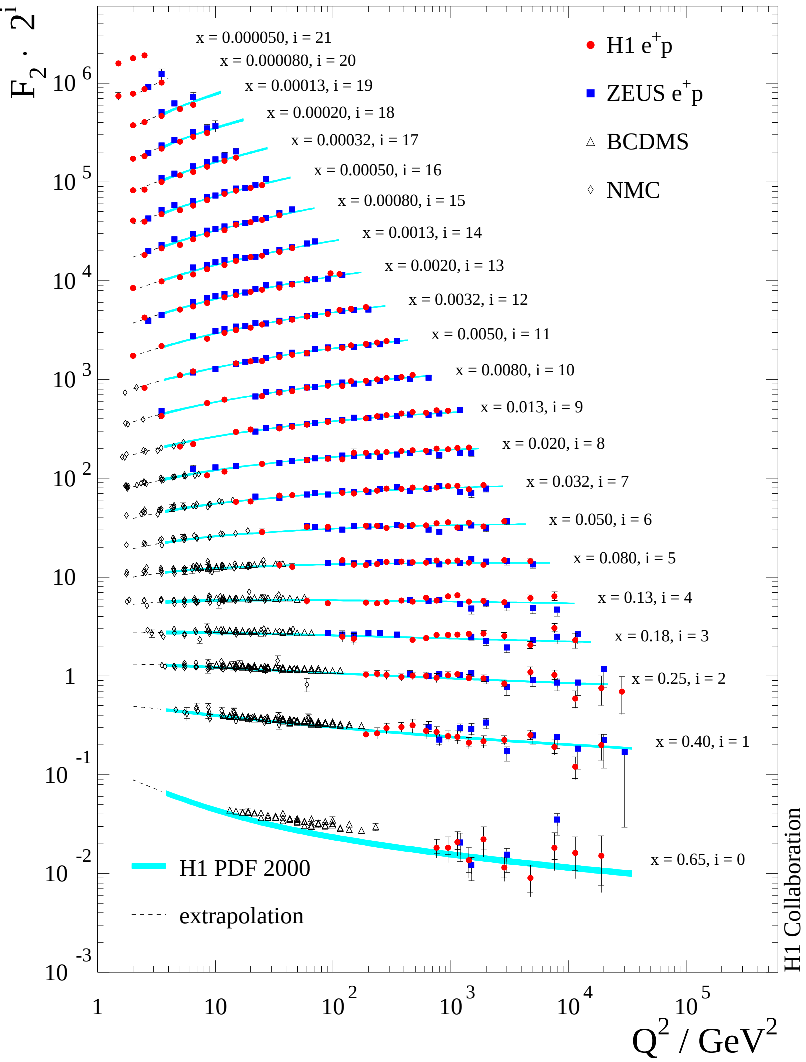
$\Longrightarrow$ **Fastest and most Precise Way of Analysis**

2. QCD Analysis of Unpolarized Parton Distributions

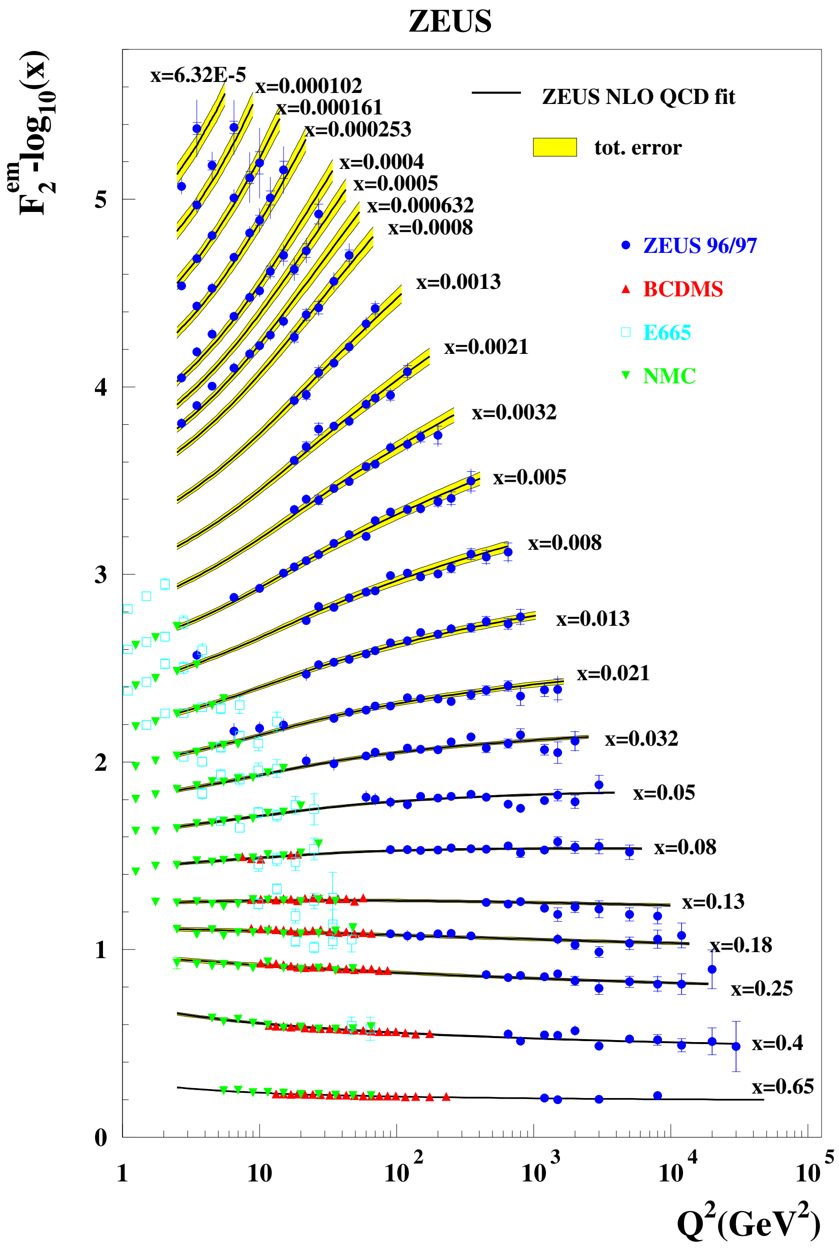
DIS range
Nucleon structure:

$10^{-5} < x < 0.9,$
$1 < Q^2 < 50.000 \mathrm{GeV}^2$



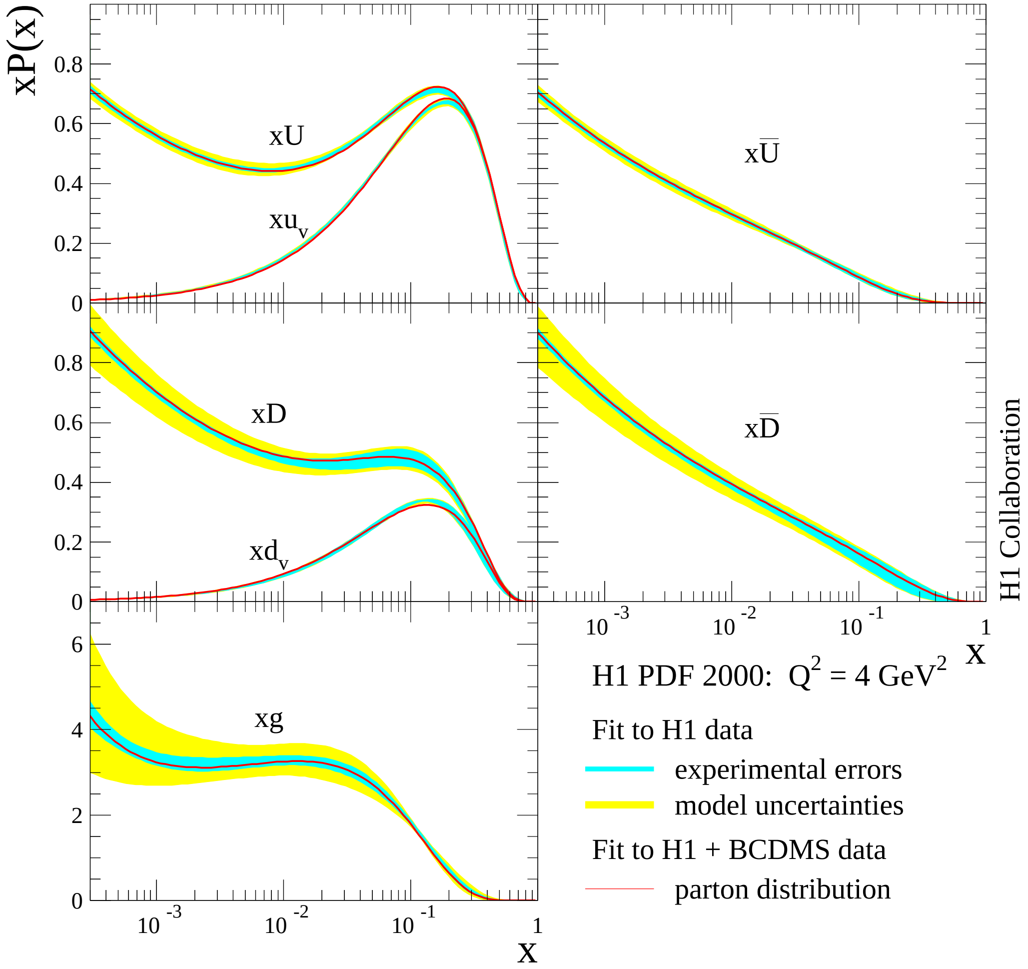


$F_2 \cdot 2^i$
H1 e+p
ZEUS e+p
BCDMS
NMC
x = 0.000050, i = 21
x = 0.000080, i = 20
x = 0.00013, i = 19
x = 0.00020, i = 18
x = 0.00032, i = 17
x = 0.00050, i = 16
x = 0.00080, i = 15
x = 0.0013, i = 14
x = 0.0020, i = 13
x = 0.0032, i = 12
x = 0.0050, i = 11
x = 0.0080, i = 10
x = 0.013, i = 9
x = 0.020, i = 8
x = 0.032, i = 7
x = 0.050, i = 6
x = 0.080, i = 5
x = 0.13, i = 4
x = 0.18, i = 3
x = 0.25, i = 2
x = 0.40, i = 1
x = 0.65, i = 0
H1 PDF 2000
extrapolation
H1 Collaboration
Q^2 / GeV^2



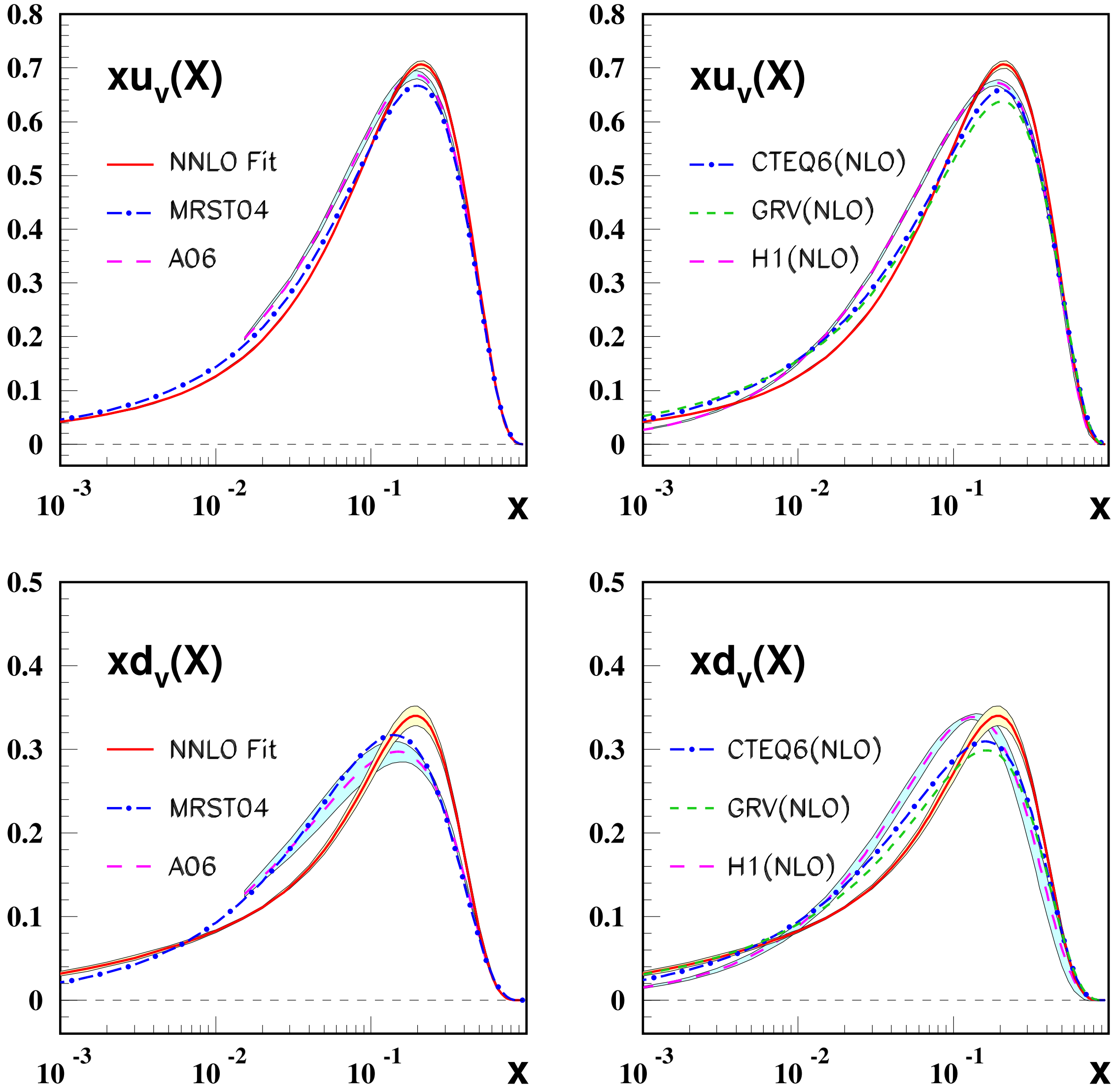
ZEUS
F_2^{em}-log$_{10}$(x)
ZEUS NLO QCD fit
tot. error
ZEUS 96/97
BCDMS
E665
NMC
x=6.32E-5
x=0.000102
x=0.000161
x=0.000253
x=0.0004
x=0.0005
x=0.000632
x=0.0008
x=0.0013
x=0.0021
x=0.0032
x=0.005
x=0.008
x=0.013
x=0.021
x=0.032
x=0.05
x=0.08
x=0.13
x=0.18
x=0.25
x=0.4
x=0.65
Q^2(GeV^2)

Parton Distributions: Overview



H1

World Data Analysis: Valence Distributions



World data:
NS-analysis

$W^2 > 12.5$ GeV^2, $Q^2 > 4$ GeV^2

N^{3}LO :

$$\alpha_s(M_Z^2) = 0.1141^{+0.0020}_{-0.0022}$$

J.B., H. Böttcher,
A. Guffanti,
(hep-ph/0607200)

Why an $O(\alpha_s^4)$ analysis can be performed?

assume an $\pm 100\%$ error on the Pade approximant $\longrightarrow$ ± 2 MeV in Λ_{QCD}

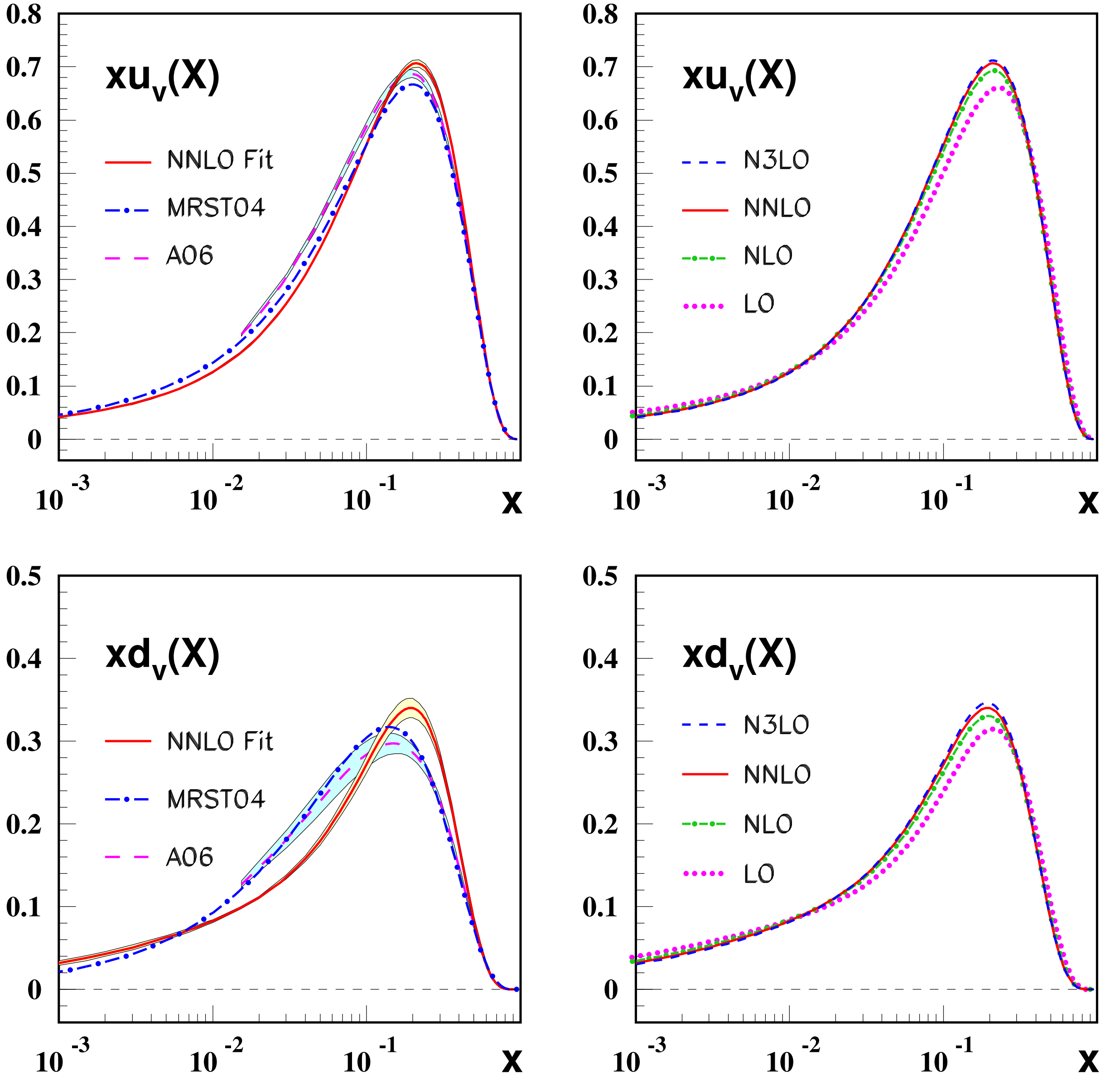
$$\gamma_n^{approx:3} = \frac{{\gamma_n^{(2)}}^2}{\gamma_n^{(1)}}$$

Baikov & Chetyrkin, April 2006:

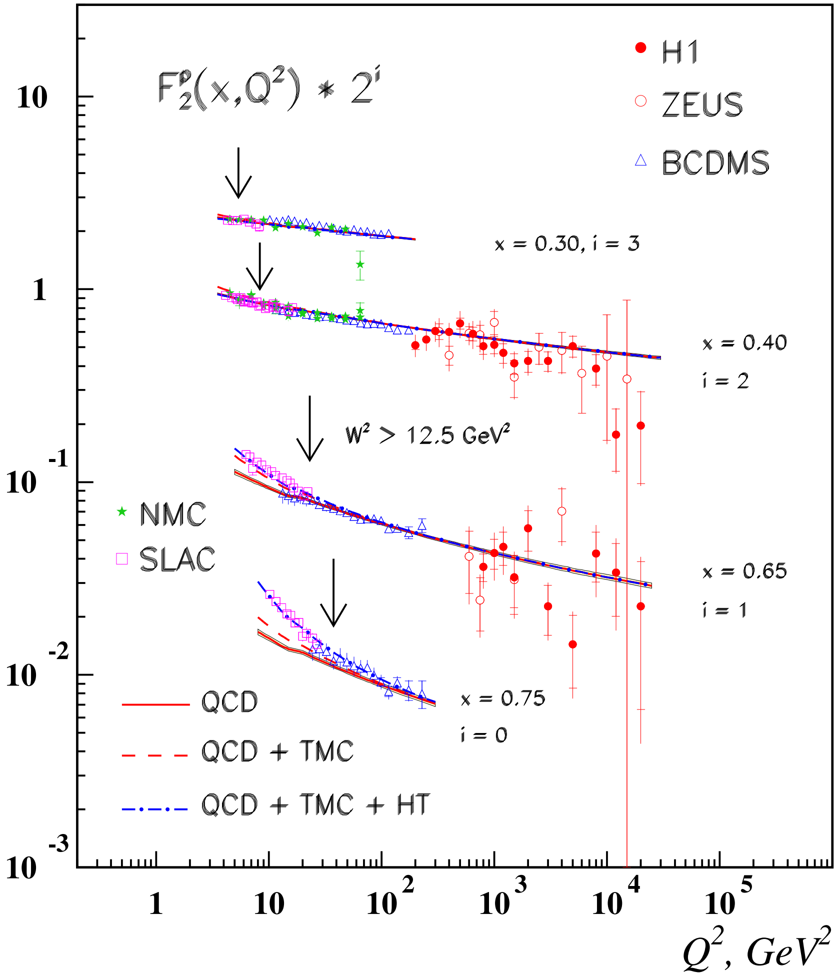
$$\begin{aligned}\gamma_2^{3;NS} &= \frac{32}{9} a_s + \frac{9440}{243} a_s^2 + \left[\frac{3936832}{6561} - \frac{10240}{81}\zeta_3\right] a_s^3 \\ &+ \left[\frac{1680283336}{1777147} - \frac{24873952}{6561}\zeta_3 + \frac{5120}{3}\zeta_4 - \frac{56969}{243}\zeta_5\right] a_s^4\end{aligned}$$

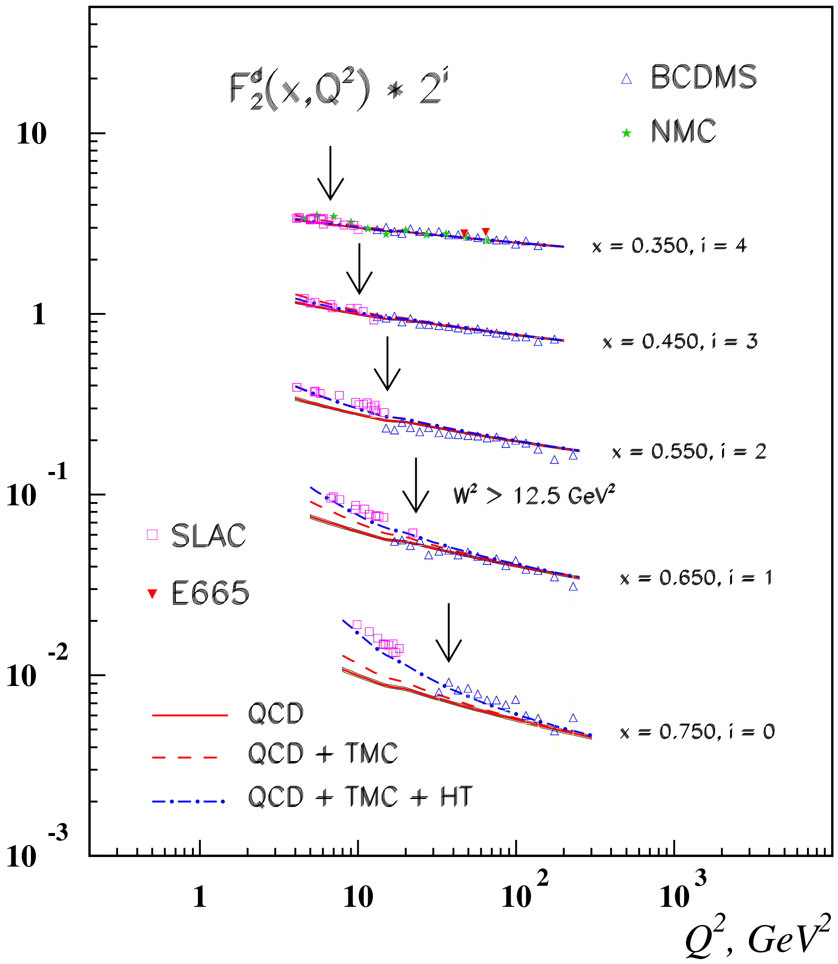
The results agree better than 20%.

Valence Distributions

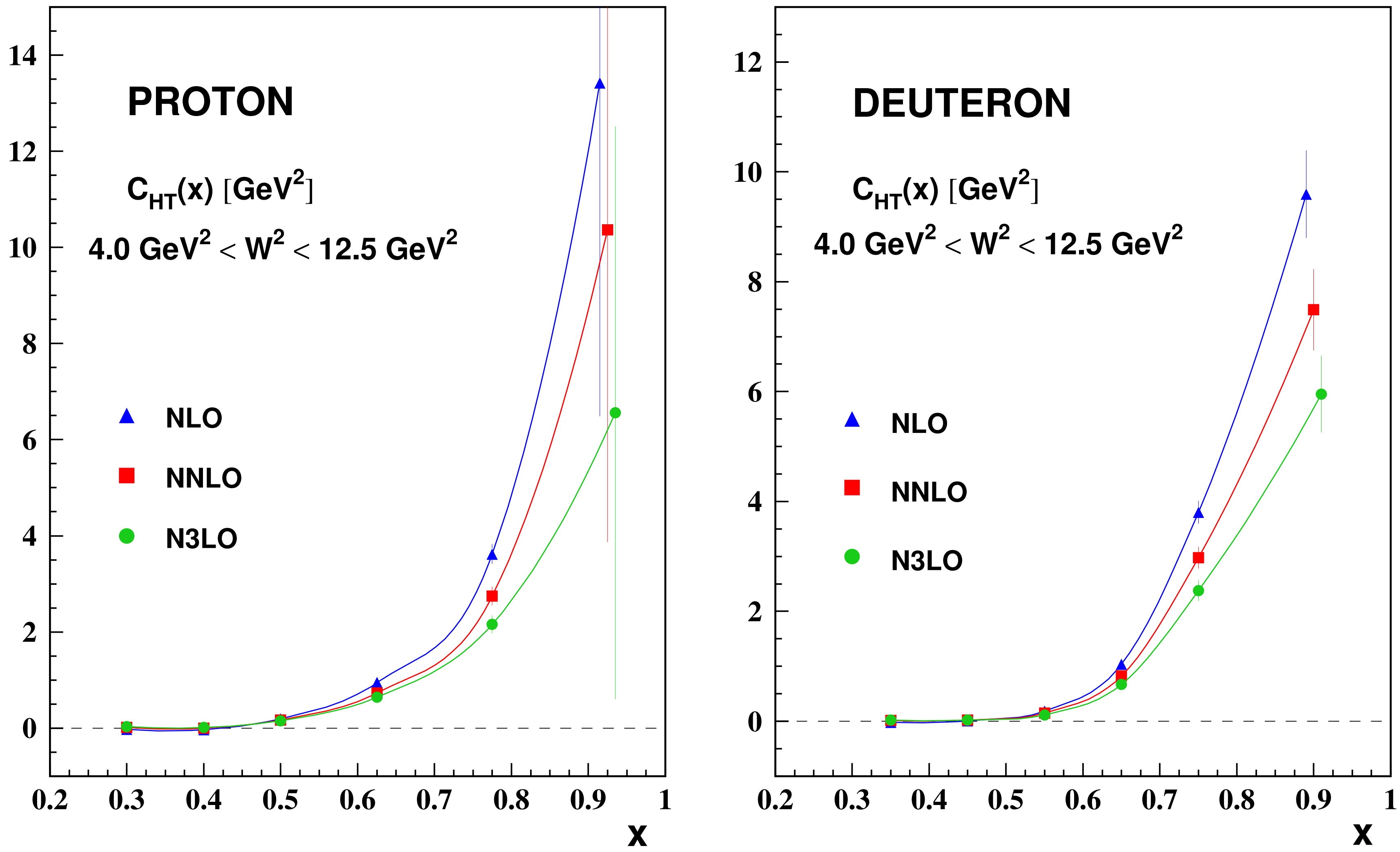


Valence Distributions



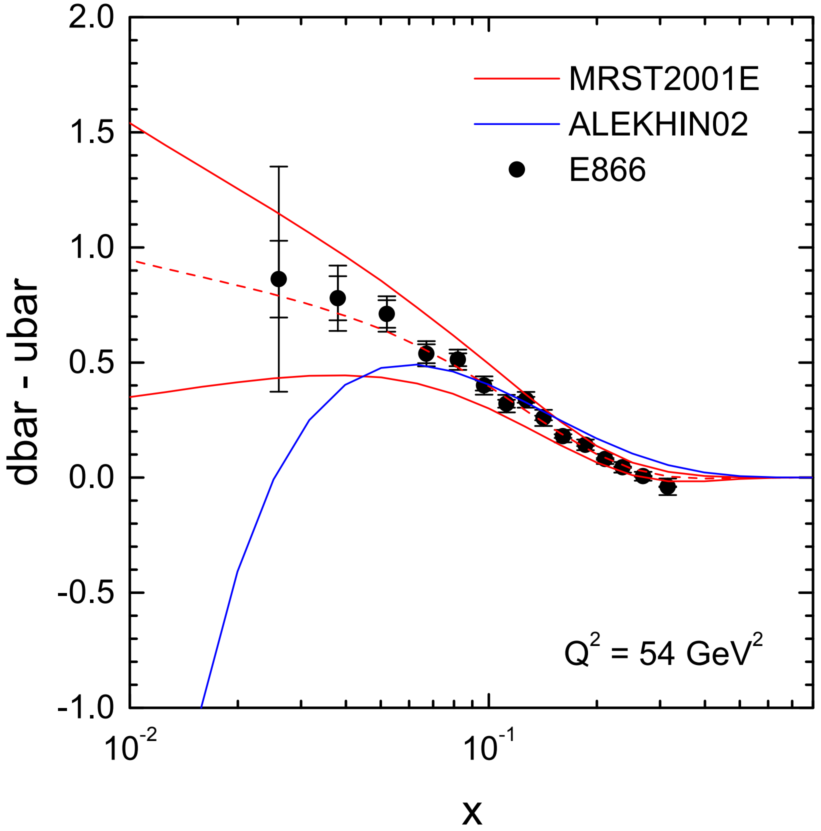


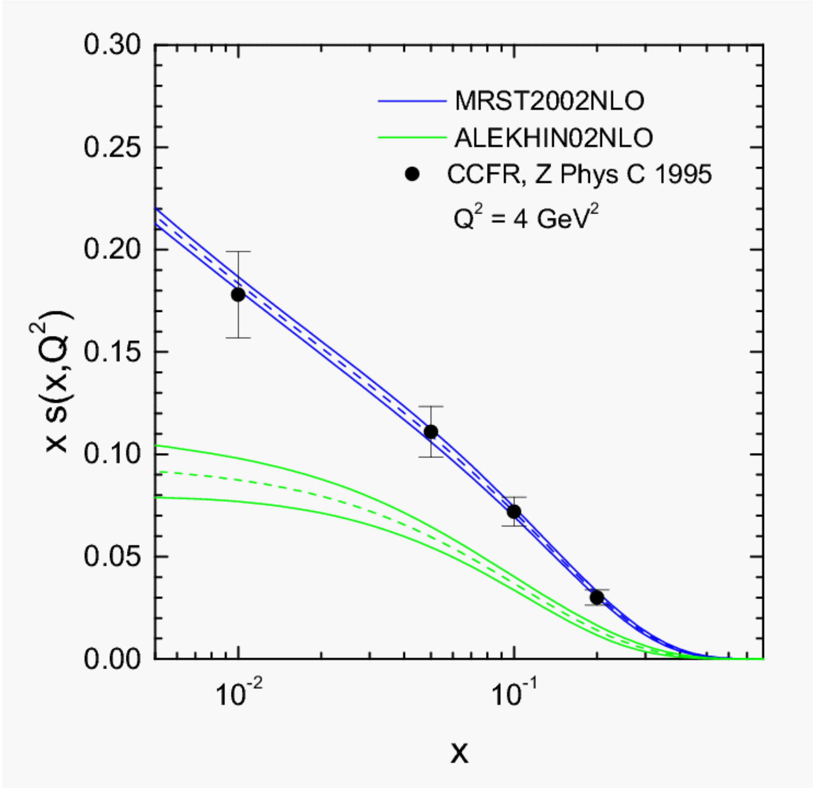
Valence Distributions: higher twist



- agreement between p and d analysis
- LGT determination of interest

Flavor distributions: light quarks



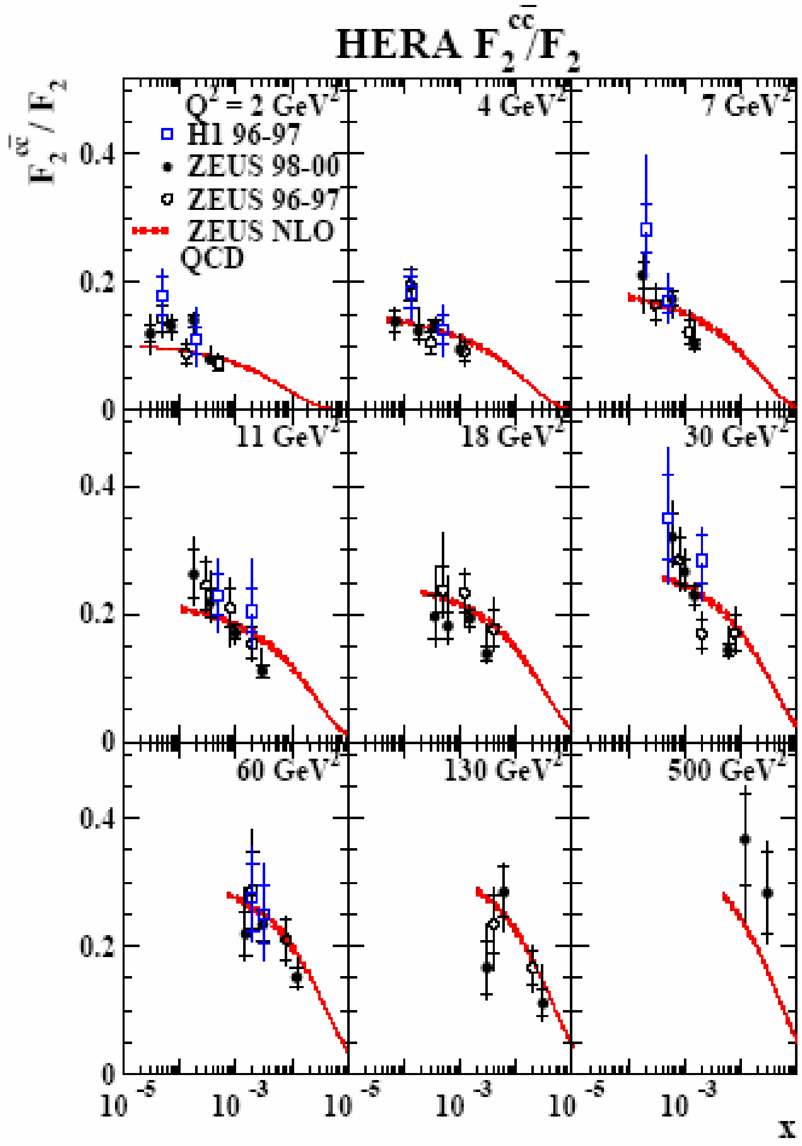


J. Stirling, 2004

More work needed.

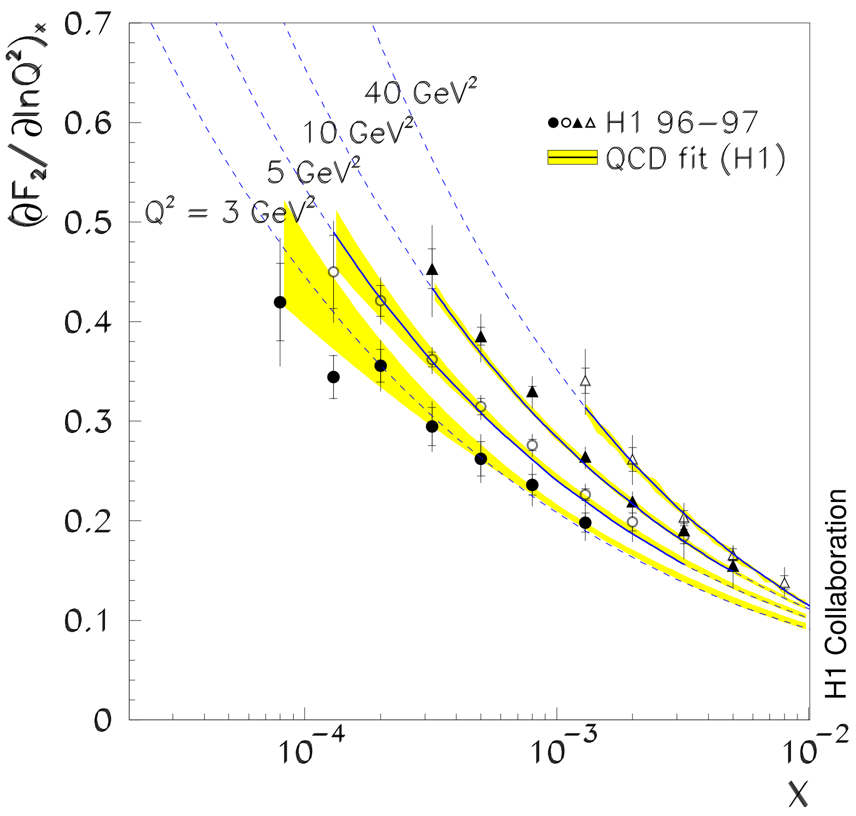
HERMES probably could measure $s(x,Q^2)$ in an independent way.

Charm

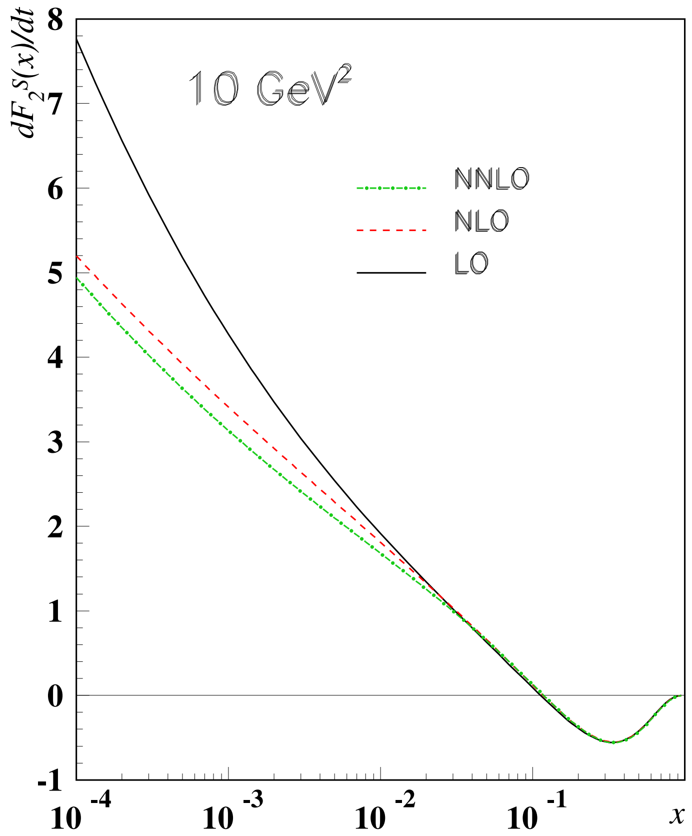


$F_2^{c\bar{c}}(x,Q^2)$ will be very well measured at HERA.

Slope of F_2 at low x



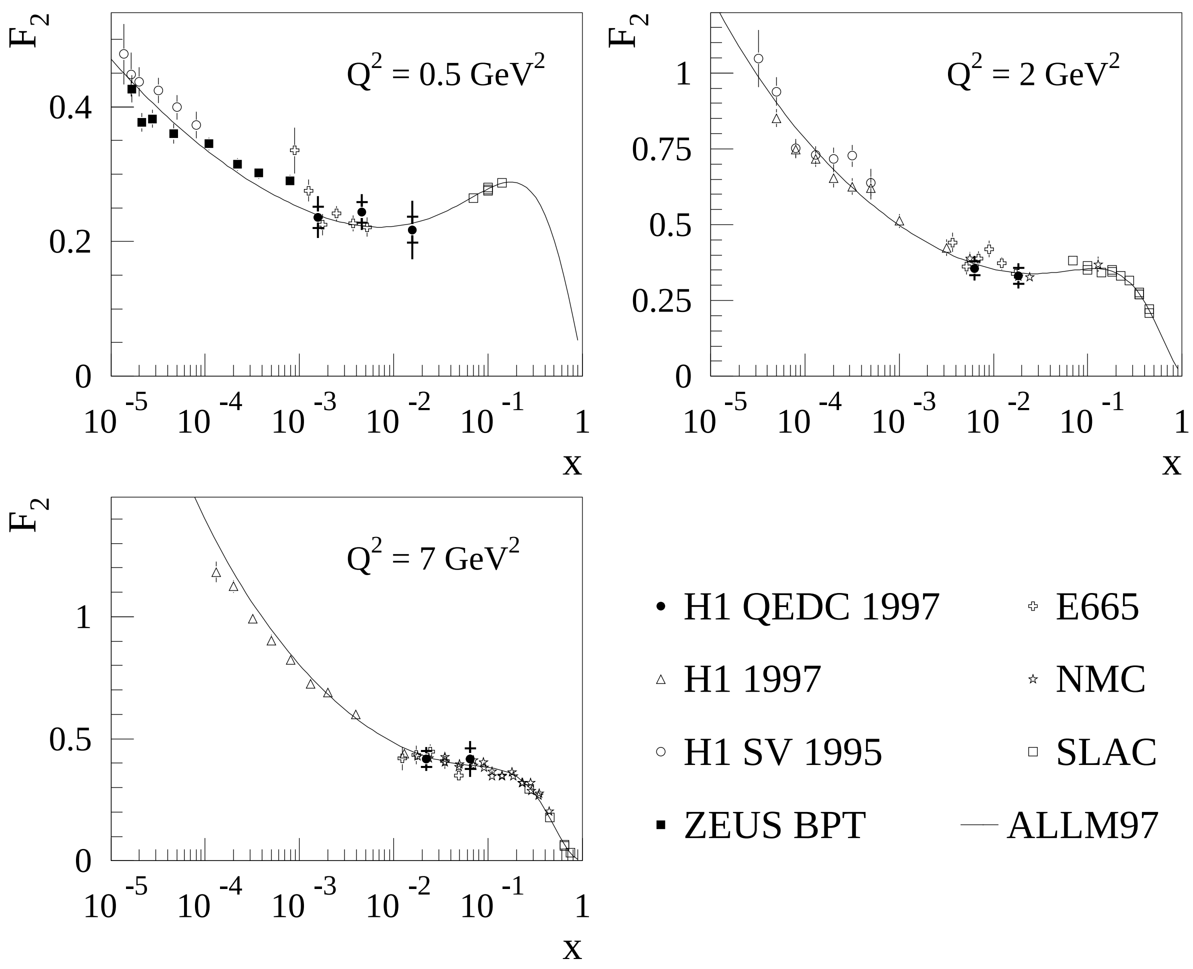
H1



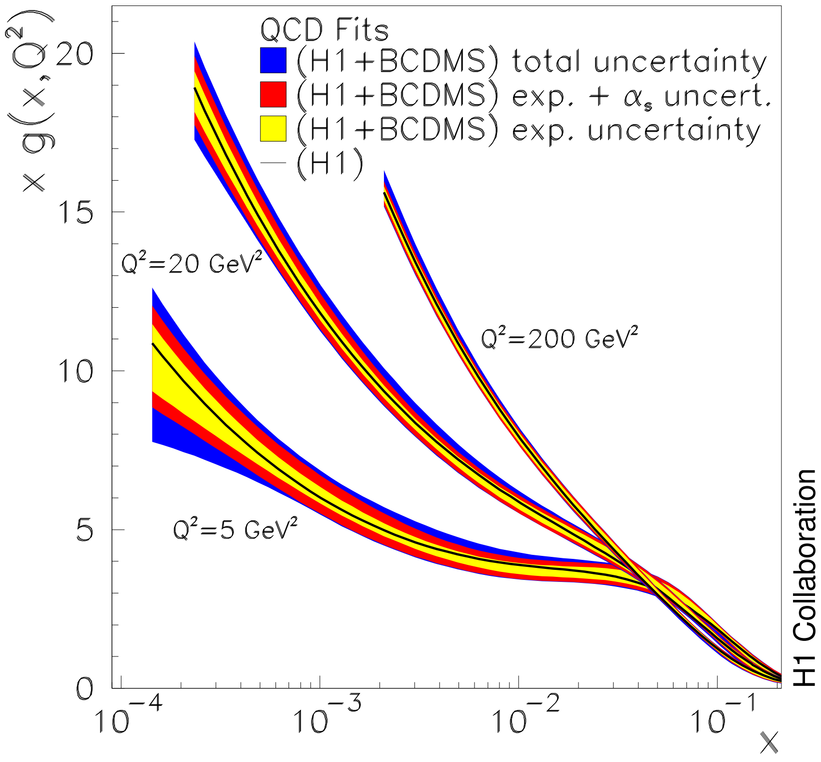
J.B., A. Guffanti 2005

Very likely, that the $\overline{\rm MS}$–gluon is remains positive!

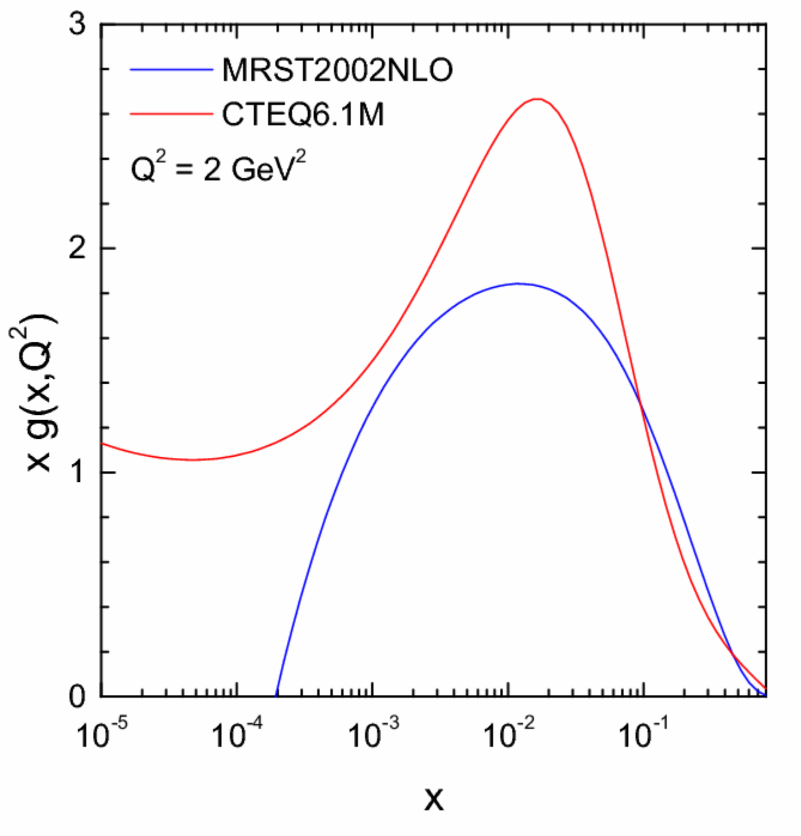
Perturbative or non-perturbative growth?



Gluon Density



H1

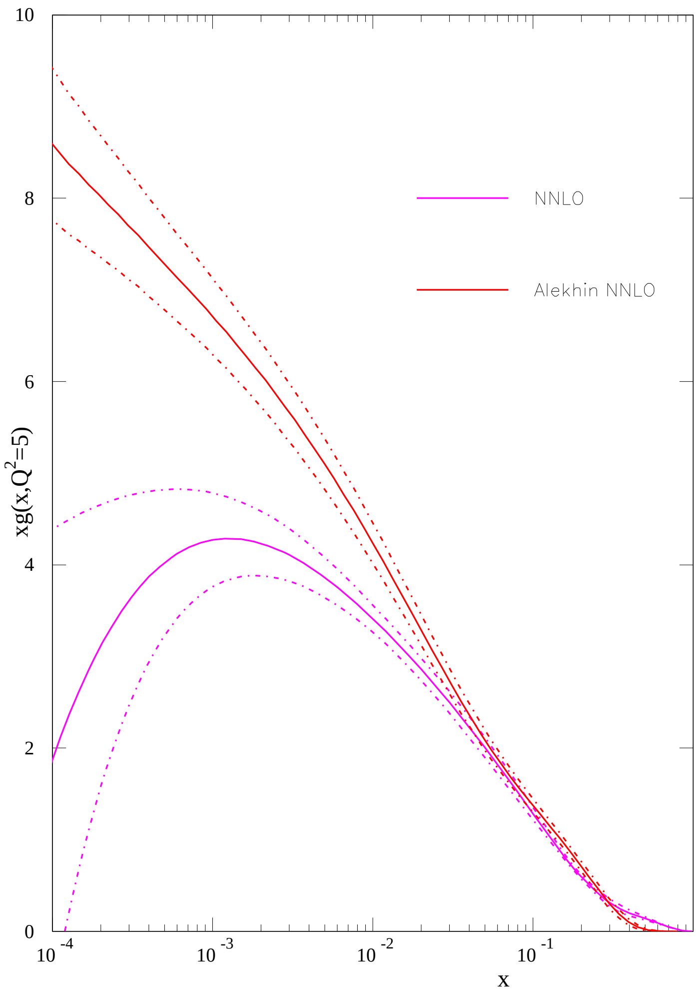


MRST 02 vs CTEQ 6

More work needed; MS– vs scheme-invariant evolution.

$F_L(x,Q^2)$ could be decisive.

Gluon Density



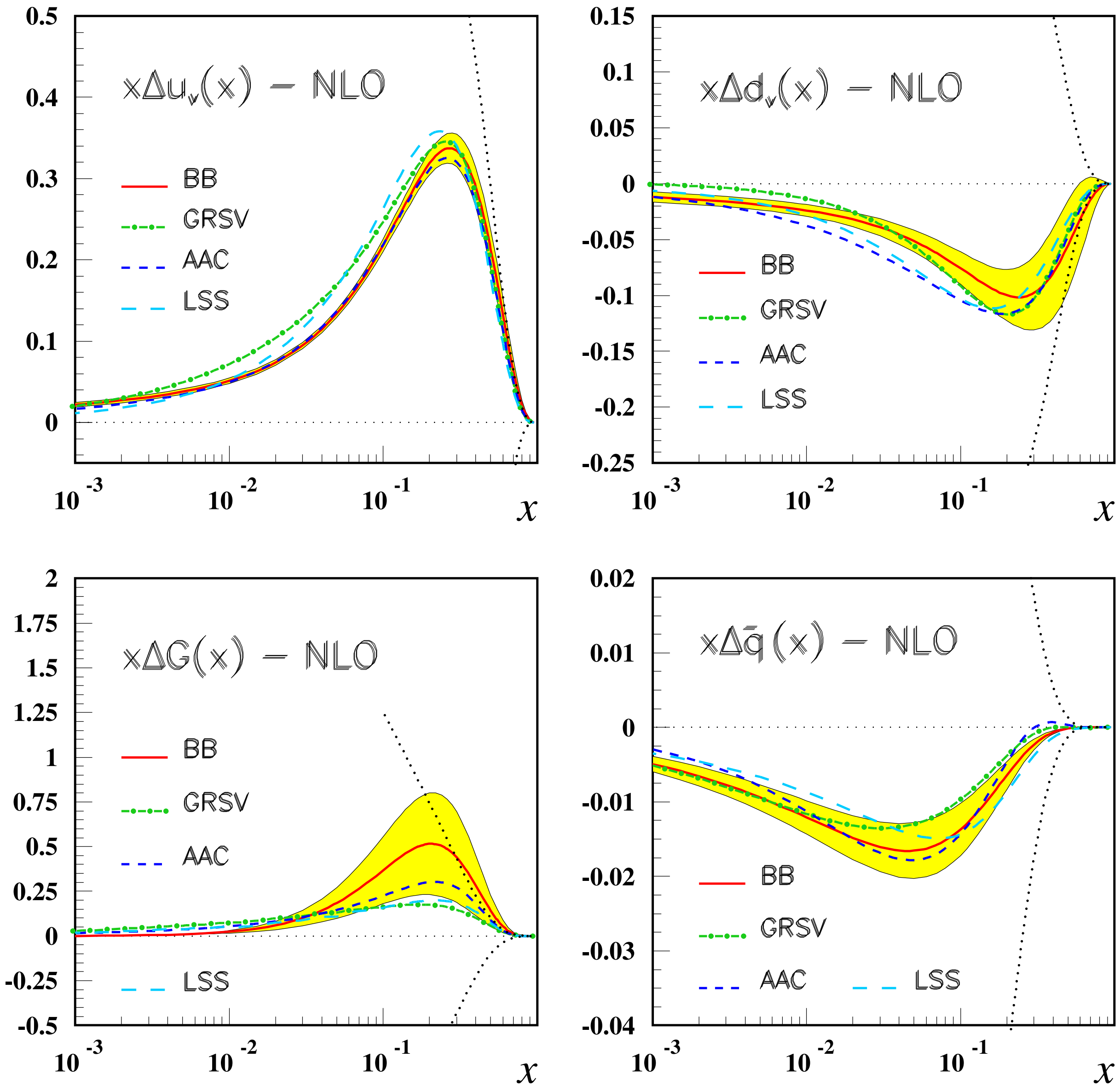
MRST06 vs Alekhin: 2006

Not both distributions can be correct.

$F_L(x, Q^2)$ could be decisive.

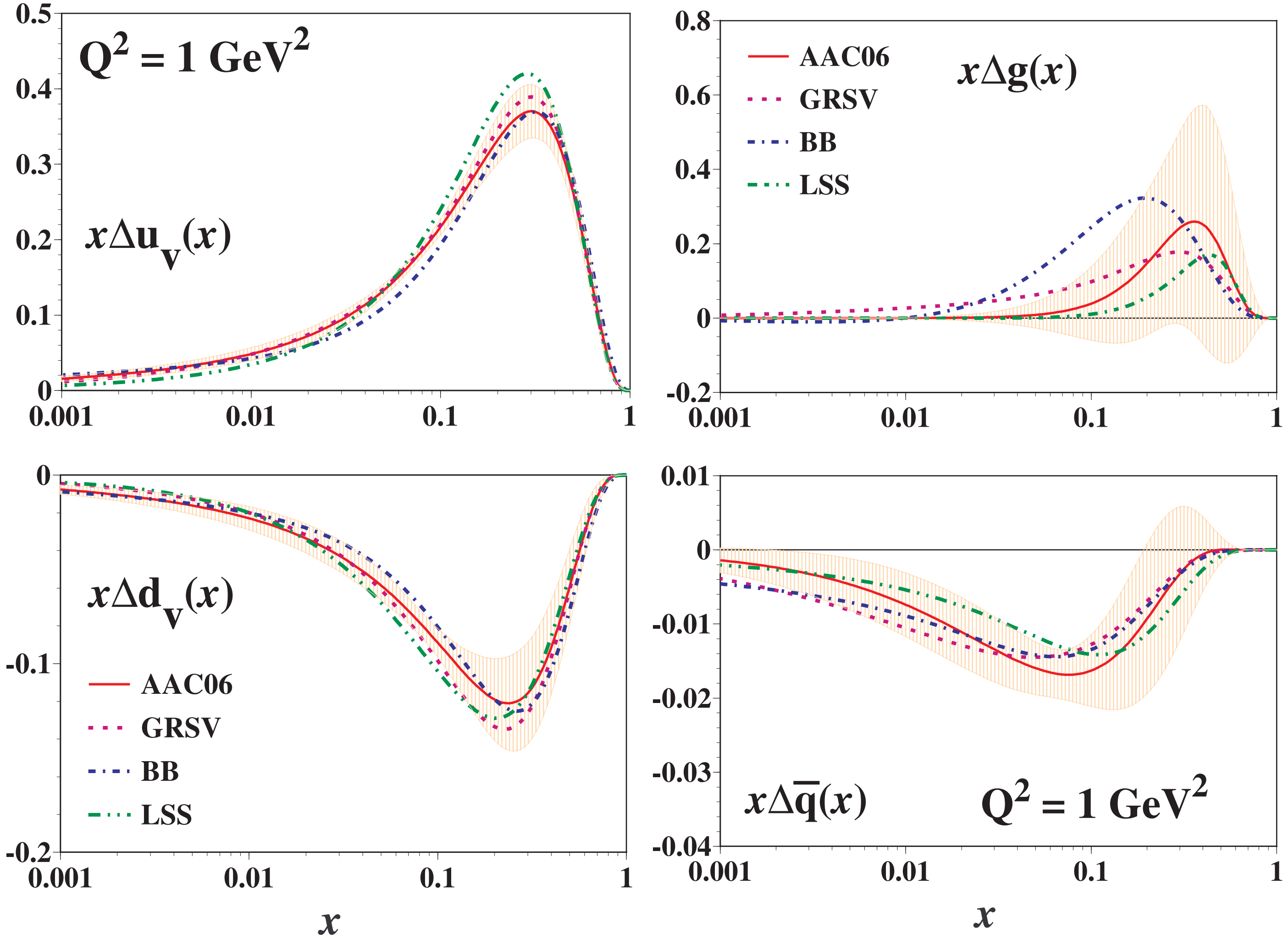
More work needed ! BBG Analysis in progress.

3. QCD Analysis of Polarized Parton Distributions



J.B., H. Böttcher, 2002

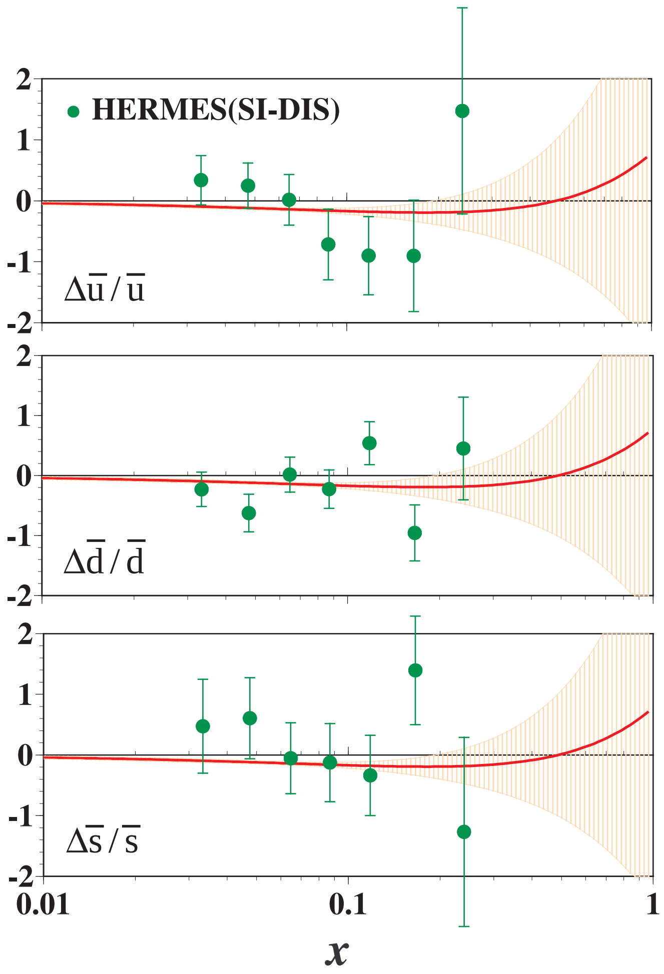
Polarized Parton Densities

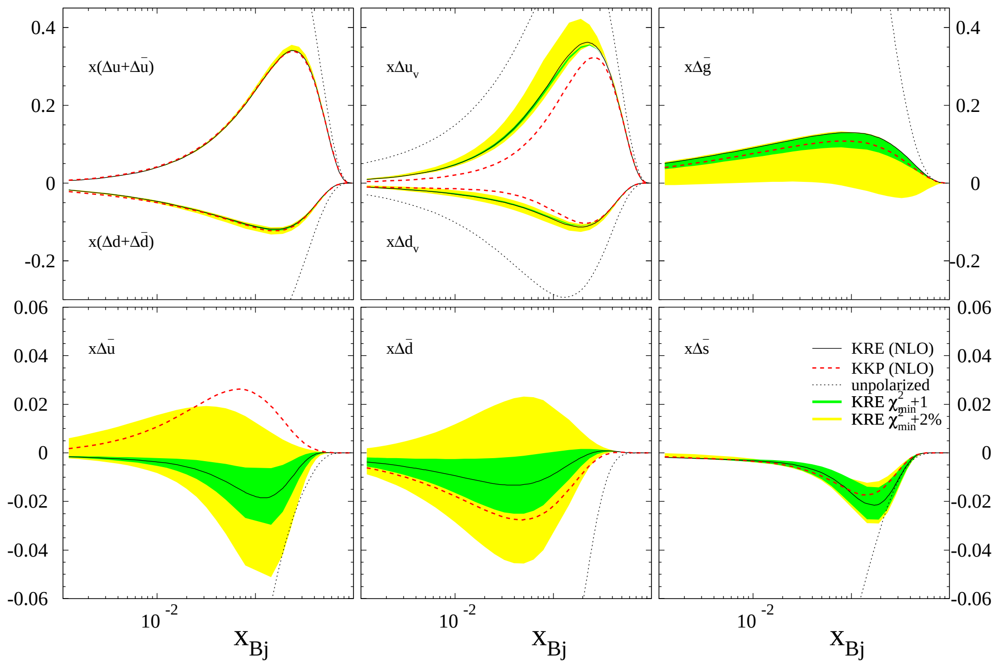


AAC 2006

Neural Networks: L. Del Debbio & A. Guffanti

Polarized Parton Densities: Flavor Separation





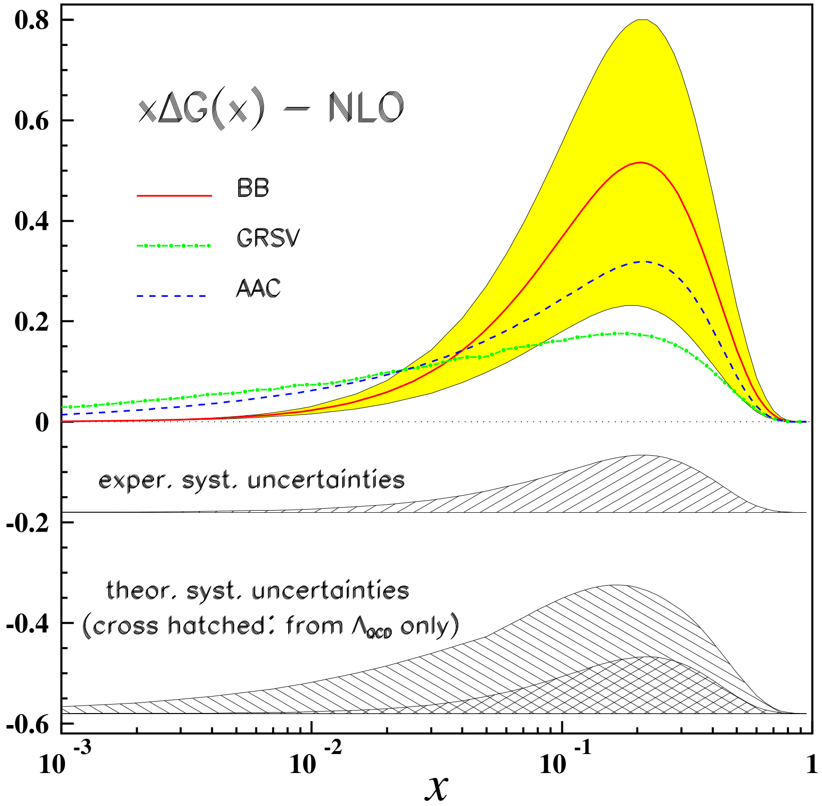
De Florian & Sassot, 2005

$\delta\Delta s$ too small ?

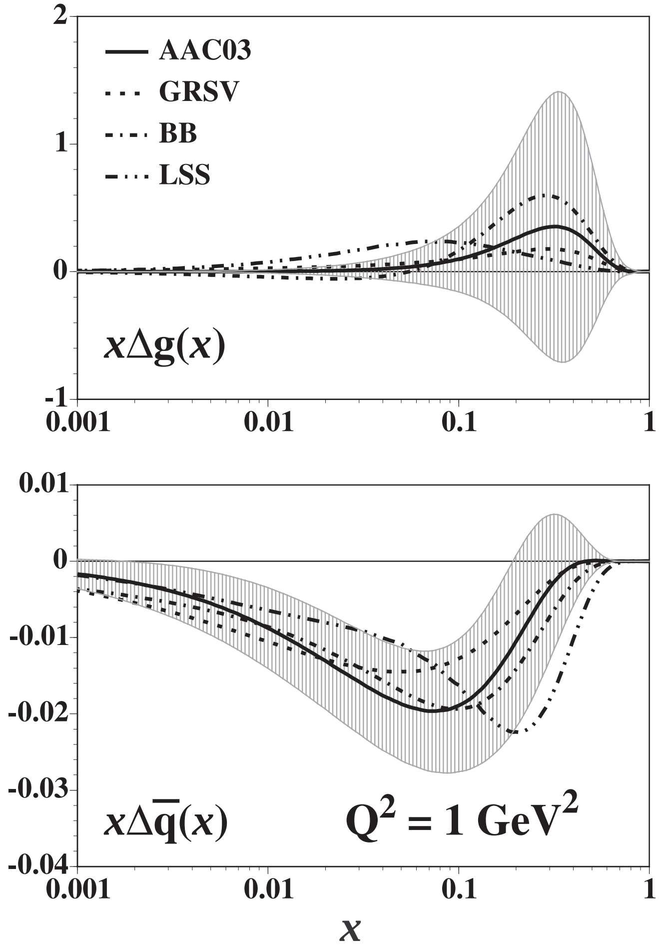
HERMES & AAC

cf. also: Y. Imazu, T. Shibata, Y. Miyachi, SPIN 2006.

Polarized Gluon Density



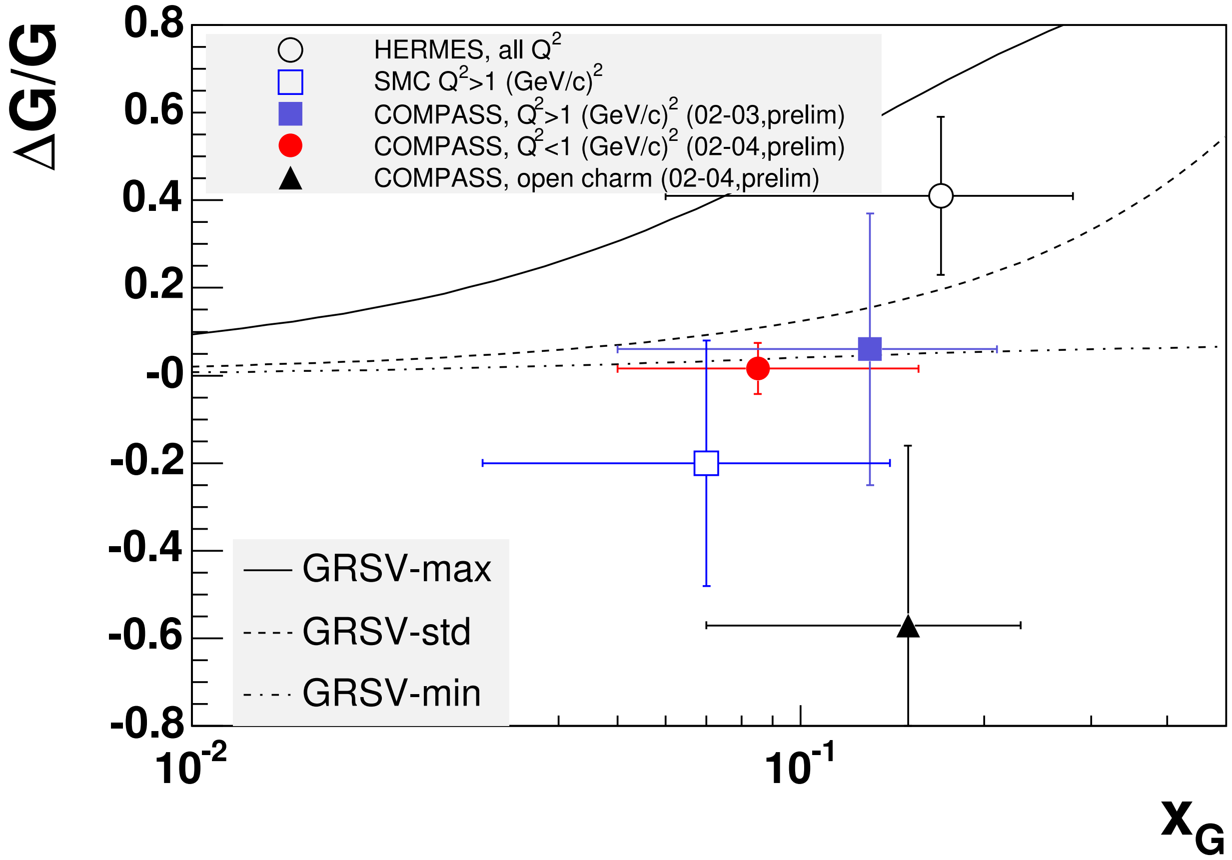
J.B., H. Böttcher, 2002



AAC

$\Longrightarrow$ Currently slight move towards lower values.

Polarized Gluon Density



COMPASS 2006 compared to other measurements

$\Longrightarrow$ Rather low Q^2; cf. G. Mallot and also P. Liebing (HERMES) SPIN 2006.

More Investigations Needed !

Moments of PDF's: PT + data

f	n	This Fit N^{3}LO	MRST04 NNLO	A02 NNLO
u_v	2	0.3006 ± 0.0031	0.285	0.304
	3	0.0877 ± 0.0012	0.082	0.087
	4	0.0335 ± 0.0006	0.032	0.033
d_v	2	0.1252 ± 0.0027	0.115	0.120
	3	0.0318 ± 0.0009	0.028	0.028
	4	0.0106 ± 0.0004	0.009	0.010
$u_v - d_v$	2	0.1754 ± 0.0041	0.171	0.184
	3	0.0559 ± 0.0015	0.055	0.059
	4	0.0229 ± 0.0007	0.022	0.024

J.B., H. Böttcher, A. Guffanti, 2006

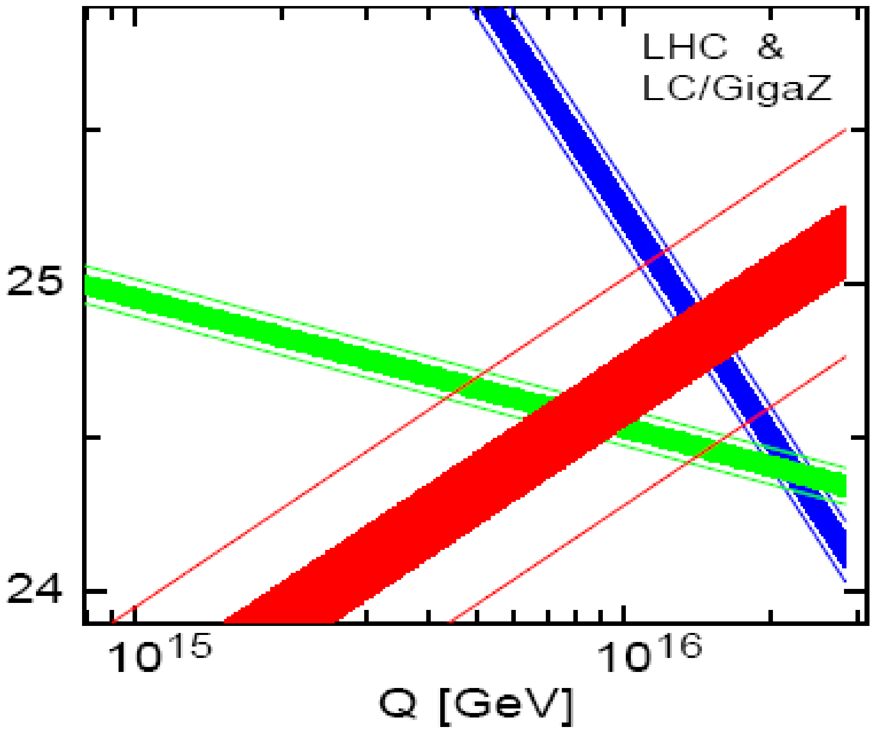
	Moment	BB, NLO
Δu_v	0	0.926
	1	0.163 ± 0.014
	2	0.055 ± 0.006
Δd_v	0	-0.341
	1	-0.047 ± 0.021
	2	-0.015 ± 0.009
$\Delta u_v - \Delta d_v$	0	1.267
	1	0.210 ± 0.025
	2	0.070 ± 0.011

J.B., H. Böttcher, 2002

Lattice Results : developping; different fermion-types studied.
Low values of m_π crucial; values approach 270 MeV now.

4. Λ_{QCD} and $\alpha_s(M_Z^2)$

$$\frac{\delta\alpha_{\rm em}(0)}{\alpha_{\rm em}(0)} \sim 3\cdot 10^{-11} \qquad \frac{\delta\alpha_{\rm weak}}{\alpha_{\rm weak}} \sim 7\cdot 10^{-4} \qquad \frac{\delta\alpha_s(M_Z^2)}{\alpha_s(M_Z^2)} > 2\cdot 10^{-2}$$



P. Zerwas, 2004

Overview of the Analyses

- Various NLO analyses; $\Rightarrow$ Precision requires NNLO analysis and higher!
- Mixed S- and NS-NNLO analyses $e(\mu)N$ world data
- S- and NS-NNLO moment analyses νN world data
- NS-N^{3}LO analysis $e(\mu)N$ world data
- NLO analyses polarized $e(\mu)N$ world data
- Lattice measurements

$\alpha_s(M_Z^2)$

NLO	$\alpha_s(M_Z^2)$	expt	theory	Ref.
CTEQ6	0.1165	± 0.0065		[1]
MRST03	0.1165	± 0.0020	± 0.0030	[2]
A02	0.1171	± 0.0015	± 0.0033	[3]
ZEUS	0.1166	± 0.0049		[4]
H1	0.1150	± 0.0017	± 0.0050	[5]
BCDMS	0.110	± 0.006		[6]
GRS	0.112			[10]
BBG	0.1148	± 0.0019		[9]
BB (pol)	0.113	± 0.004	${}^{+0.009}_{-0.006}$	[7]

NLO

NNLO	$\alpha_s(M_Z^2)$	expt	theory	Ref.
MRST03	0.1153	± 0.0020	± 0.0030	[2]
A02	0.1143	± 0.0014	± 0.0009	[3]
SY01(ep)	0.1166	± 0.0013		[8]
SY01(νN)	0.1153	± 0.0063		[8]
GRS	0.111			[10]
A06	0.1128	± 0.0015		[11]
BBG	0.1134	$+0.0019/-0.0021$		[9]

N^{3}LO	$\alpha_s(M_Z^2)$	expt	theory	Ref.
BBG	0.1141	$+0.0020/-0.0022$		[9]

NNLO and N^{3}LO

BBG: $N_f = 4$: non-singlet data-analysis at $O(\alpha_s^4)$: $\Lambda = 234 \pm 26$ MeV

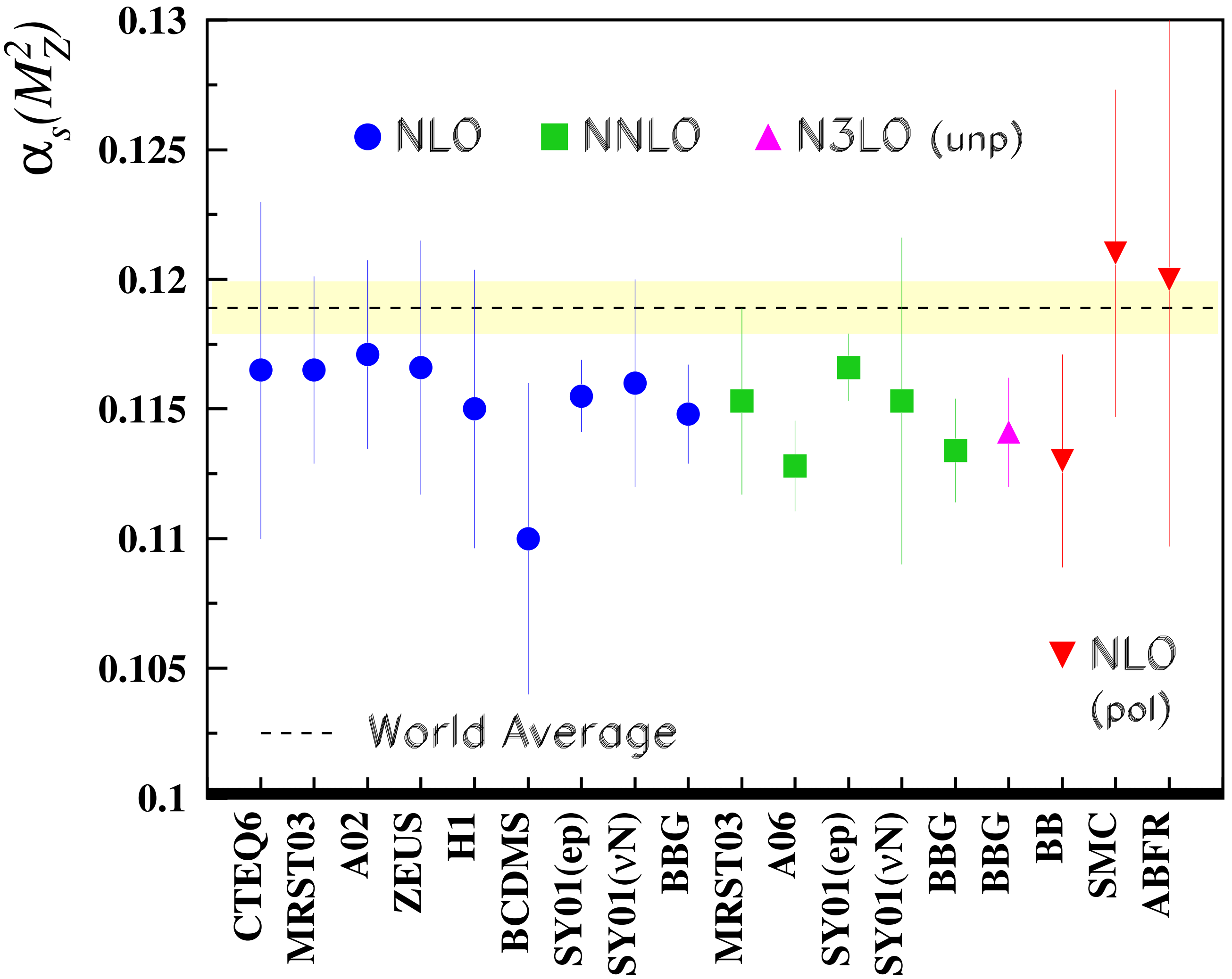
Lattice results :

Alpha Collab: $N_f = 2$ Lattice; non-pert. renormalization $\Lambda = 245 \pm 16 \pm 16$ MeV

QCDSF Collab: $N_f = 2$ Lattice, pert. reno. $\Lambda = 261 \pm 17 \pm 26$ MeV

Lepage et al.: Larger Values, to be discussed.

$\alpha_s(M_Z^2)$



J.B., H. Böttcher, A. Guffanti, 2006

5. The Needs : What would we like to know ?

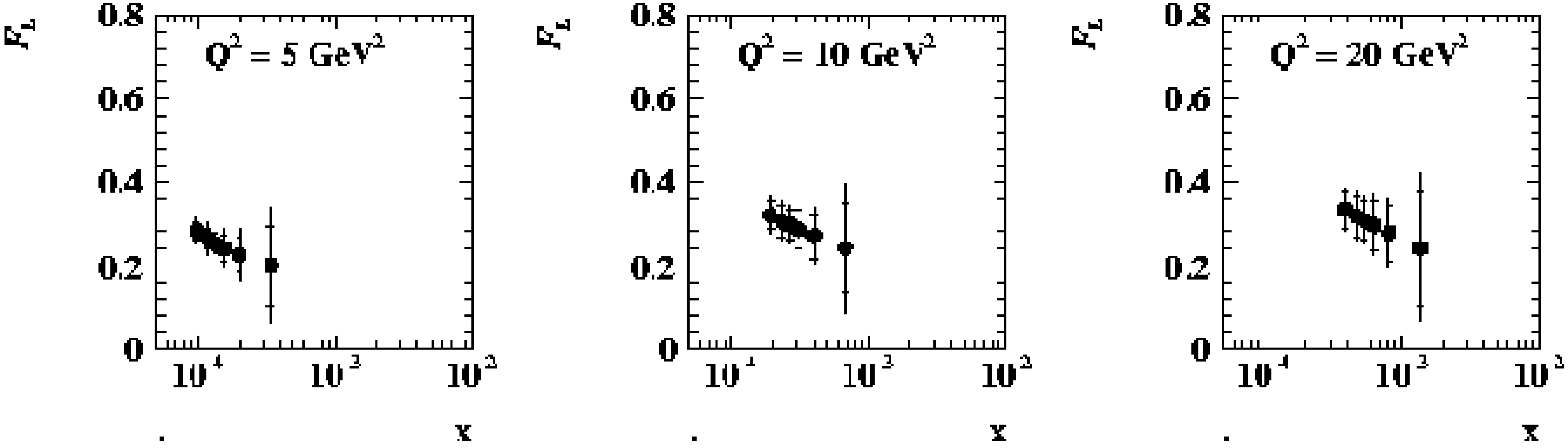
HERA:

- Collect high luminosity for $F_2(x,Q^2)$, $F_2^{c\bar{c}}(x,Q^2)$, $g_2^{c\bar{c}}(x,Q^2)$, and measure $h_1(x,Q^2)$.
- Measure : $F_L(x,Q^2)$. This is a key-question for HERA.

$F_L(x, Q^2)$

M. Klein, 2004: Projection for a possible measurement at HERA

$\Longrightarrow$ of central importance to study the small x behaviour of the gluon distribution



4. Future Avenues : What would we like to know ?

HERA:

- Collect high luminosity for $F_2(x,Q^2)$, $F_2^{c\bar{c}}(x,Q^2)$, $g_2^{c\bar{c}}(x,Q^2)$, and measure $h_1(x,Q^2)$.
- Measure : $F_L(x,Q^2)$. This is a key-question for HERA.

RHIC & LHC:

- Improve constraints on gluon and sea–quarks: polarized and unpolarized. DIS PDF's $\Longleftrightarrow$ Collider PDF's

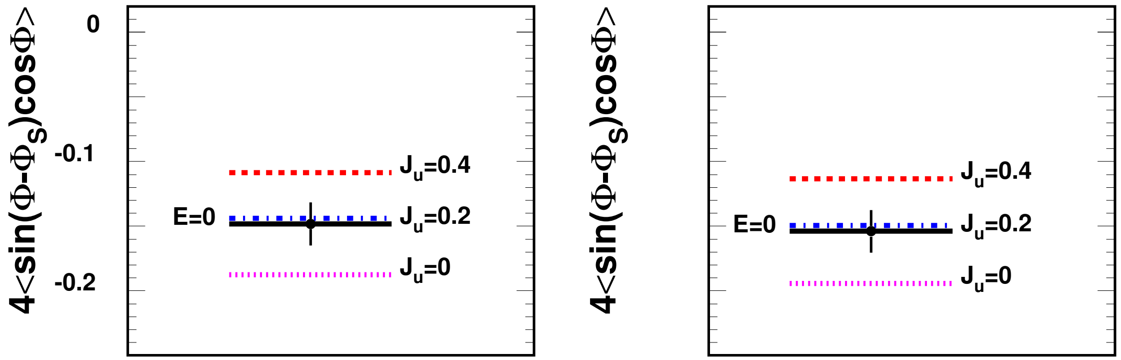
JLAB:

- High precision measurements in the large x domain at unpolarized and polarized targets; supplements HERA's high precision measurements at small x.

L_q from DVCS

- HERA and JLAB : Improve DVCS data

Theory widely developed, cf. rev. Belitsky & Radyushkin, 2005



Expected DVCS asymmetry $A_{UT}^{\sin(\phi-\phi_S)\cos\phi}$ with $b_v = 1$, $b_s = \infty$, $J_u = 0.4(0.2, 0.0)$, $J_d = 0.0$ in the Regge (left panel) and factorized (right panel) ansatz, at the average kinematics of the full measurement. $E = 0$ denotes zero effective contribution from the GPD E. The projected statistical error for 8M DIS events is shown. The systematic error is expected to not exceed the statistical one.

F. Ellinghaus et al. 2005

The measurement of L_q off data is model–dependent at the moment.
Lattice calculations at low pion masses are needed to complete the picture

Graph Resummation and Saturation

Further study of proposed mechanisms needed: RHIC, LHC for nucleus-nucleus collisions.

ep scattering: partly different mechanisms

more studies would be welcome; link to higher twist contributions in gluon–dynamics

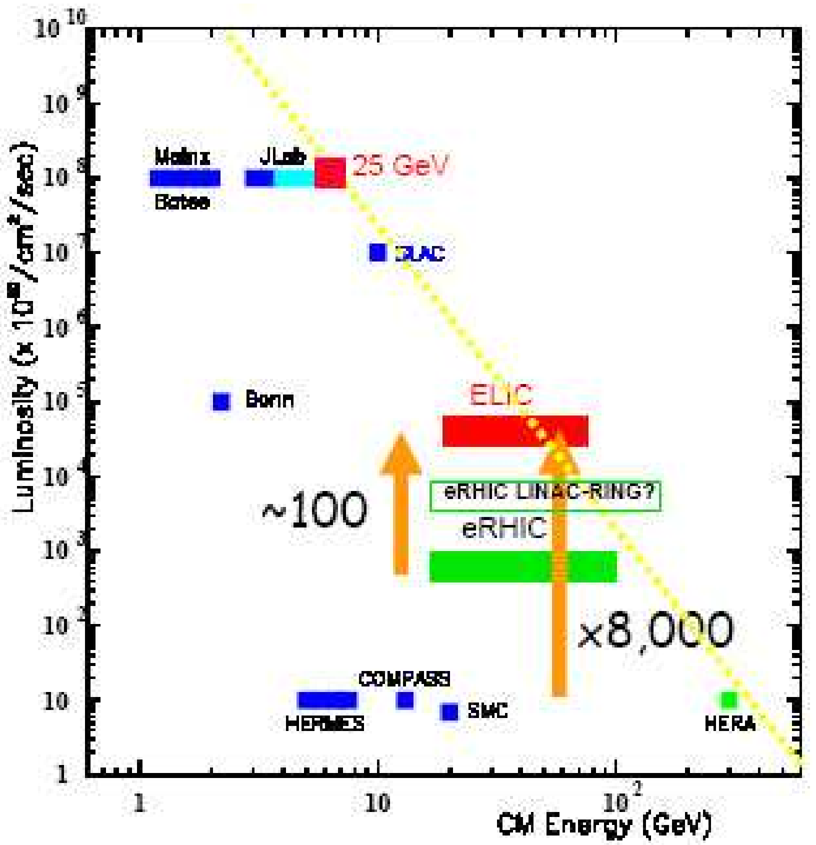
How do the non-perturbative and perturbative parts factorize ?

Conservation laws and interplay between the small x and medium x range behaviour

New DIS Machines

Where to go ?

- High energies : small x, large Q^2 desirable.
- High luminosities : ELIC: $\sqrt{s}$ between CERN and HERA energies



R. Ent,2004
high precision physics
polarized and unpolarized

Would be an important extension of the present programmes in many respects.

Enhancing Precision Further...

- What is the correct value of $\alpha_s(M_z^2)$? $\overline{\mathrm{MS}}$-analysis vs. scheme-invariant evolution helps. Compare non–singlet and singlet analysis; careful treatment of heavy flavor. (Theory & Experiment)
- Flavor Structure of Sea-Quarks: More studies needed.(All Experiments)
- Revisit polarized data upon arrival of the 3-loop anomalous dimensions; NLO heavy flavor contributions needed. (Theory)
- QCD at Twist 3: $g_2(x, Q^2)$, semi–exclusive Reactions, Transversity, diffraction in polarized scattering (HERMES, High Precision polarized experiments, JLAB, ELIC)
- Comparison with Lattice Results: α_s, Moments of Parton Distributions, Angular Momentum.

Enhancing Precision Further...

- Calculation of more hard scattering reactions at the 3–loop level: LHC
- Further perfection of the mathematical tools:
  $\Longrightarrow$ Algorithmic simplification of Perturbation theory in higher orders.
- Even higher order corrections needed ?