B-Meson Distribution Amplitudes

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Heavy Flavour Mini-Workshop , SFB/TR9 , Aachen, 26.10.05

Outline

- Prehistory
- B-meson DA: definition
- 3-particle DA's
- Evolution of $\phi_+^B(\omega, \mu)$
- Use of QCD sum rules
- Models of $\phi_+^B(\omega)$
- Summary on λ_B

Prehistory of B-Meson DA

- B-meson "wave function", quark models of exclusive B decays

$$\langle x_q \rangle = (m_q/m_b) \; , \quad x_q = p_q/p_B \; , \; m_q\text{-"constituent" mass}$$

. [M.Bauer,B.Stech, M.Wirbel (1985)]

- heavy-light analog of pion DA, finite m_b

[V.Chernyak, A.Zhitnisky , I.Zhitnisky (1985)]

- B-meson DA in the context of PQCD factorization for $B \to \pi$

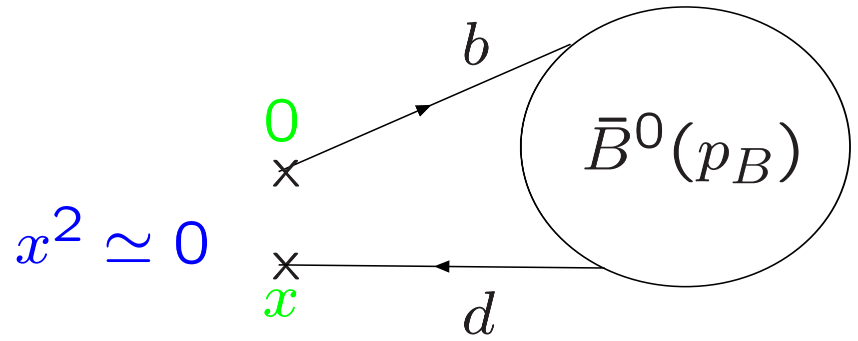
. [A. Szczepaniak, E. M. Henley and S. J. Brodsky (1990)]

see also . [R. Akhoury, G. Sterman and Y. P. Yao (1994)]

modern branch: k_t factorization in PQCD approach [H.n.Li et al]

B-Meson DA: the definition

. [A.G. Grozin. M.Neubert (1997)]



- Light-cone matrix element, consistent with HQET

$$\langle 0|T\left\{\bar{d}_\alpha(x)[x,0]b_\beta(0)\right\}|\bar{B}^0(v)\rangle|_{x^2=0}$$

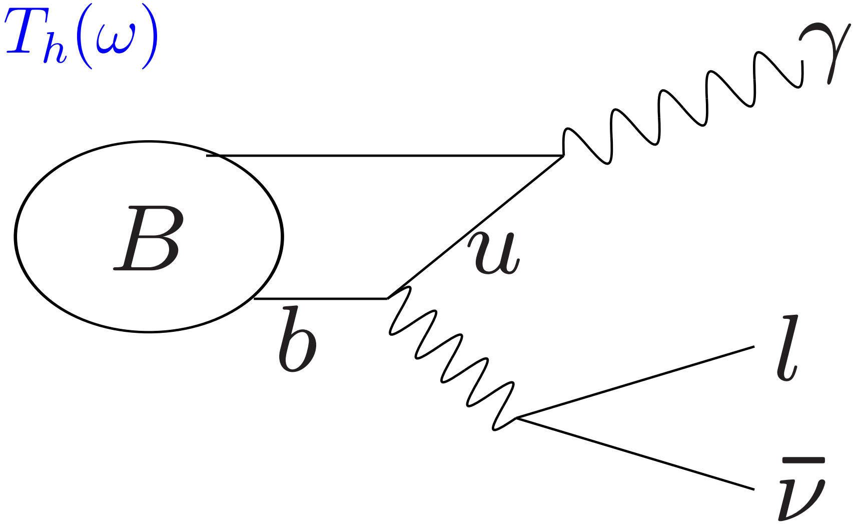
$$= -\frac{if_B m_B}{4}\left[(1+\not{v})\gamma_5\int_0^\infty d\omega e^{-i\omega v\cdot x}\left\{\phi_+^B(\omega)+\frac{\phi_+^B(\omega)-\phi_-^B(\omega)}{2v\cdot x}\not{x}\right\}\gamma_5\right]_{\beta\alpha}$$

$p_B = m_B v$, $[x,0]$-Wilson line, (scale-dependence not yet specified)

Factorization in $B \to \gamma l \nu_l$

$(p_l + p_\nu)^2 \sim 0,\ E_\gamma \sim m_B/2$

$A(B \to \gamma l \nu) \sim \int d\omega \phi_+^B(\omega) T_h(\omega)$



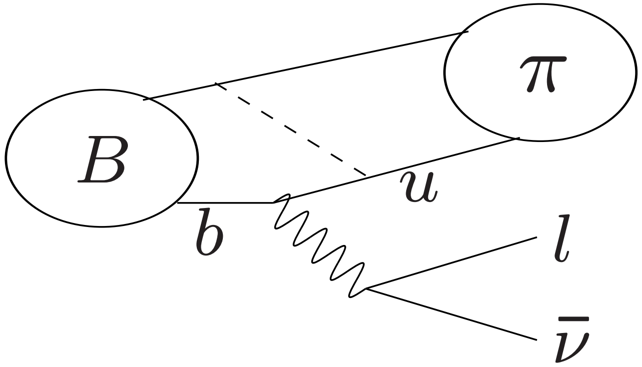
$T_h \sim 1/\omega,$

$1/\lambda_B = \int_0^\infty d\omega \frac{\phi_+^B(\omega)}{\omega}$

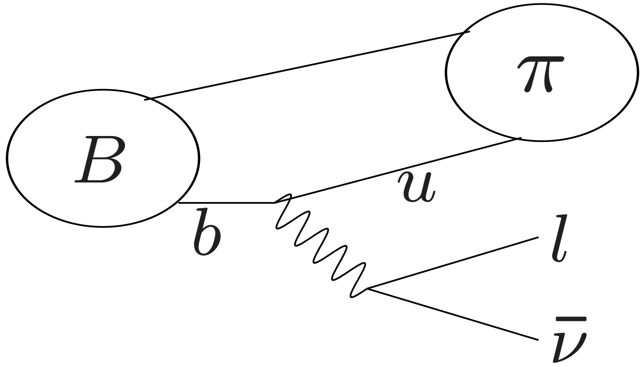
- inverse moment

G. P. Korchemsky, D. Pirjol, T. M. Yan (2000)
S. Descotes-Genon, C. T. Sachrajda (2003)
E. Lunghi, D. Pirjol, D. Wyler (2003),
S. W. Bosch, R. J. Hill, B. O. Lange, M. Neubert (2004)

B-Meson DA in $B \to \pi$



"hard", factoriz.



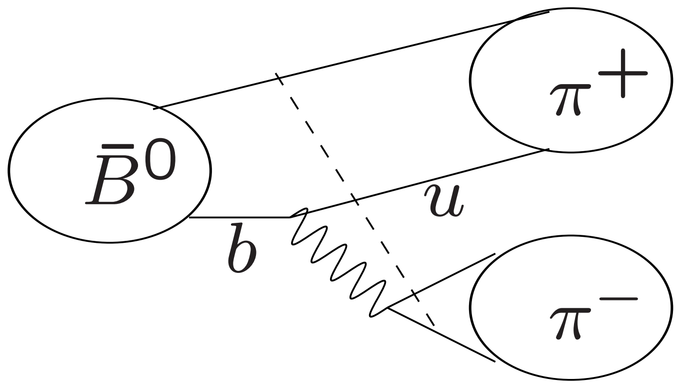
"soft", nonfact.

$$f_{B\pi}(q^2) \sim \alpha_s(\mu) \int d\omega du \phi_+^B(\omega, \mu) T_h(q^2, \omega, u, \mu) \varphi_\pi(u, \mu) + f_{B\pi}^{soft}(q^2)$$

B-meson DA enters only the hard-scattering part

M. Beneke and T. Feldmann (2000)

B-Meson DA in $B \to \pi\pi$



$$A(B \to \pi\pi) \sim C(\mu) f_\pi f_{B\pi}(0) m_B^2 + ...$$

$$+\alpha_s(\mu)\tilde{C}(\mu) \int d\omega du dv \phi_+^B(\omega,\mu) T_h(\omega,u,v,\mu)\varphi_\pi(u,\mu)\varphi_\pi(v,\mu) + ...$$

B-meson DA enters the "hard-spectator" nonfact.ampl.

M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda (1999)

more recently: B meson DA's in SCET ...

Properties of $\phi_{\pm}^{B}(\omega)$

- model-independent constraint from QCD equation of motion, Wandzura-Wilczek-type relation:

$$\phi_{-}^{B}(\omega) = \int_{\omega}^{\infty} d\rho \frac{\phi_{+}^{B}(\rho)}{\rho} \quad \Rightarrow \phi_{-}^{B}(0) = 1/\lambda_B$$

(neglecting the three-particle $b\bar{q}G$ DA's of B)

- boundary condition: $\omega \to 0$: $\phi_{+}^{B}(\omega) \sim \omega, \quad \phi_{-}^{B}(0) = const$

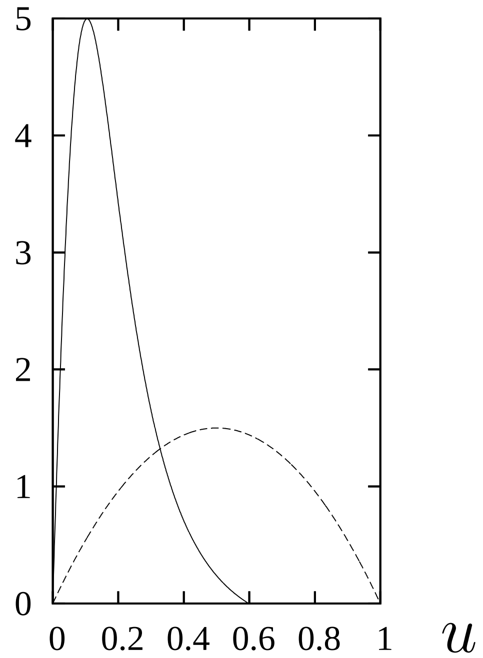
Parton interpretation of $\phi_\pm^B(\omega)$?

- $\int_0^\infty d\omega \phi_\pm^B(\omega) = 1$,
(local limit well defined, normalization $\to f_B$)

- variable $\omega = (l_0 + l_3)$:
(l-the spectator quark momentum
in B rest frame)

- heavy-light kinematics:
$\phi_\pm^B(\omega) \neq 0$ at $0 < \omega \leq 2\bar{\Lambda}$, $(\bar{\Lambda} = m_B - m_b)$

- typical $\varphi_+^B(u)$ $(u = \omega/m_B)$ vs $\varphi_\pi(u) \sim u(1-u)$

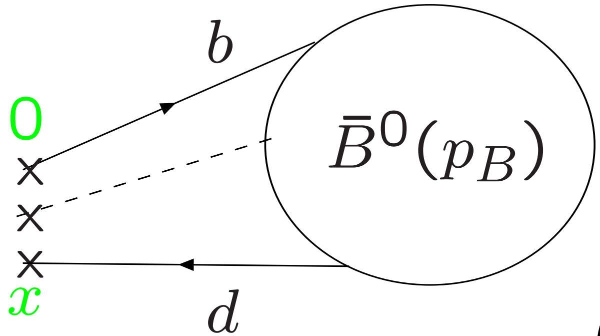


Caution: no QCD radiative corrections/renormalization yet!

Quark-antiquark-gluon DA's: definition

[H. Kawamura, J. Kodaira, C.F.Qiao and K. Tanaka,(2001)]

$x^2 \simeq 0$



$$\langle 0|\bar{d}_\alpha(x) G_{\lambda\rho}(ux) b_\beta(0)|\bar{B}^0(v)\rangle = \frac{f_B m_B}{4}\int_0^\infty d\omega \int_0^\infty d\xi\, e^{-i(\omega+u\xi)v\cdot x}$$

$$\times\Bigg[(1+\not{v})\Bigg\{(v_\lambda\gamma_\rho - v_\rho\gamma_\lambda)\Big(\Psi_A(\omega,\xi)-\Psi_V(\omega,\xi)\Big) - i\sigma_{\lambda\rho}\Psi_V(\omega,\xi)$$

$$-\left(\frac{x_\lambda v_\rho - x_\rho v_\lambda}{v\cdot x}\right) X_A(\omega,\xi) + \left(\frac{x_\lambda \gamma_\rho - x_\rho \gamma_\lambda}{v\cdot x}\right) Y_A(\omega,\xi)\Bigg\}\Bigg]_{\beta\alpha}.$$

Quark-antiquark-gluon DA's: what do we know ?

- related to $\phi_{\pm}^{B}(\omega)$ via QCD equations of motion

- modified WW relation, schematically:
$\phi_{-}^{B}(0) = 1/\lambda_B + \int d\omega d\xi \{\Psi_{V,A}(\omega,\xi)\ \},$

- behavior at small ω, ξ

- normalization of $q\bar{q}G$ DA's related
to the first and second moments of $\phi_{\pm}^{B}(\omega)$

Evolution of ϕ_+^B

- calculable in HQET [M. Neubert, B. Lange, (2003)]
- one-loop renormalization of the light-cone operator

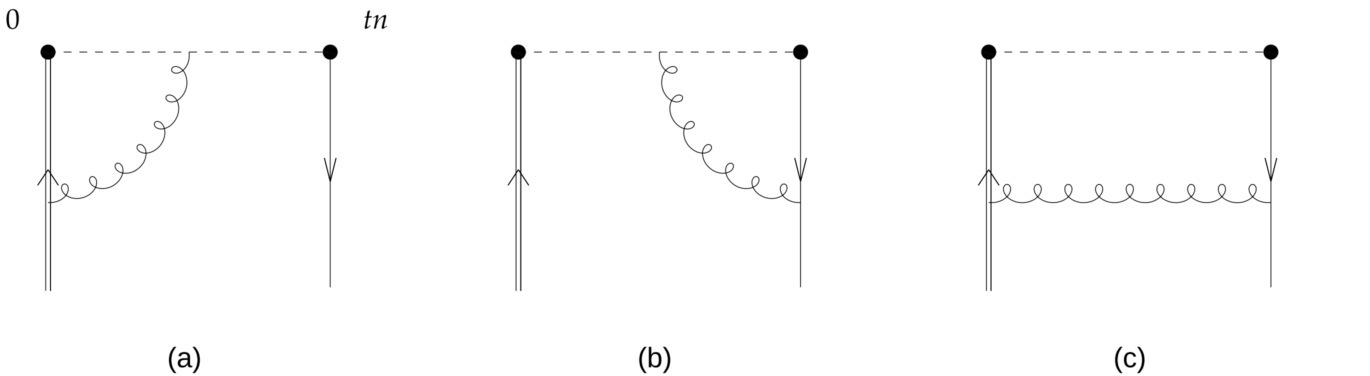
$$O_+(t) = \bar{q}(tn)\, \not{n}\, [tn, 0]\Gamma\, h_v(0).$$

v_μ-heavy quark velocity, $n^2 = 0$, $n \cdot v = 1$; light-like interval $x = tn$

$$\langle 0|O_+(t)|B_v\rangle \sim \Phi_+^B(t) = \int_0^\infty d\omega e^{-i\omega t}\phi_+^B(\omega)$$

- hard gluon exchange $\longrightarrow$ UV divergences, $O_+^{bare} \longrightarrow O_+^{ren}$

Renormalization



$$O_+^{ren}(t) = O_+^{bare}(t)\left(1 + \frac{\alpha_s C_F}{4\pi}\left[\frac{4}{\epsilon^2} + \frac{4}{\epsilon}\log(i\mu t) + ...\right]\right)$$

. –diag.(a)—

• the heavy-light vertex correction diverges at $t \to 0$ ($x \to 0$), troublesome ? the "cusp anomalous dimension"

$\phi_(\omega, \mu)$ after renormalizaiton

- expansion in local operators not possible $O_+^{ren}(t) \neq \sum_n^\infty t^n O_n(0)^{ren}$
- positive moments $\int_0^\infty d\omega \omega^N \phi_+^B(\omega, \mu)$, $N \geq 0$ divergent
including the normalization to f_B
- no parton interpretation for $\phi_+^B(\omega, \mu)$

- but ! no problem for factorization theorems
containing the inverse moment λ_B

- evolution equation for λ_B

$$\mu \frac{d}{d\mu}(\lambda_B(\mu))^{-1} = 2\alpha_s/\pi(\lambda_B(\mu))^{-1} + 4\alpha_s/\pi \int_0^\infty d\omega \frac{\phi_+^B(\omega,\mu)}{\mu} \ln(\omega/\mu)$$

UV cutoff and first two moments

[S.J.Lee and M.Neubert,hep-ph/0509350]

- introduce UV cutoff , calculate first two moments in $O(\alpha_s)$

  $M_N(\Lambda_{UV}, \mu) = \int_0^{\Lambda_{UV}} d\omega \; \omega^N \phi_+^B(\omega, \mu)$

  $M_0 = 1 + \frac{\alpha_s C_F}{4\pi} f_0(\ln(\Lambda_{UV}/\mu), \bar{\Lambda}/\Lambda_{UV})$

  $M_1 = 4\bar{\Lambda}/3(1 + \frac{\alpha_s C_F}{4\pi} \ldots.)$

- model-independent prediction for the radiative tail at one-loop:

  $\phi_+^B(\omega, \mu) \sim -\frac{\alpha_s C_F}{\pi} \ln(\omega/\mu)/\omega$

QCD sum rules for ϕ^B

- in HQET, use the Chernyak-Zhitnisky method for Gegenbauer moments of φ_π [A. G. Grozin and M. Neubert (1997)]

- recent NLO calculation (including radiative corrections!) [V. M. Braun, D. Y. Ivanov and G. P. Korchemsky,(2003)]

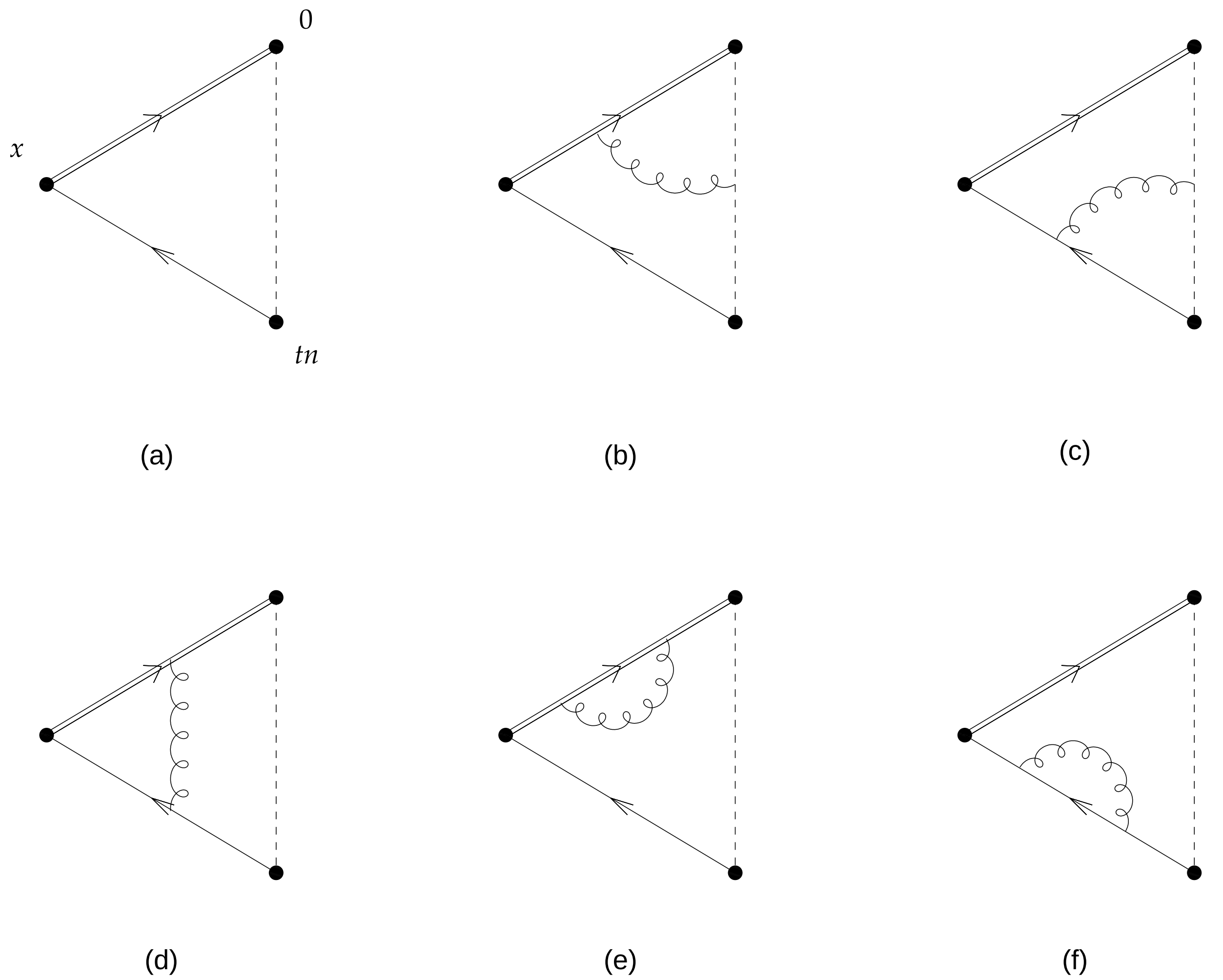
- The correlator:

$$i\int d^4x\, e^{-ik(vx)}\langle 0|\mathrm{T}\{O_+(t)\bar{h}_v(x)\Gamma_2 q(x)\}|0\rangle = \{...\}T(t,k)\,.$$

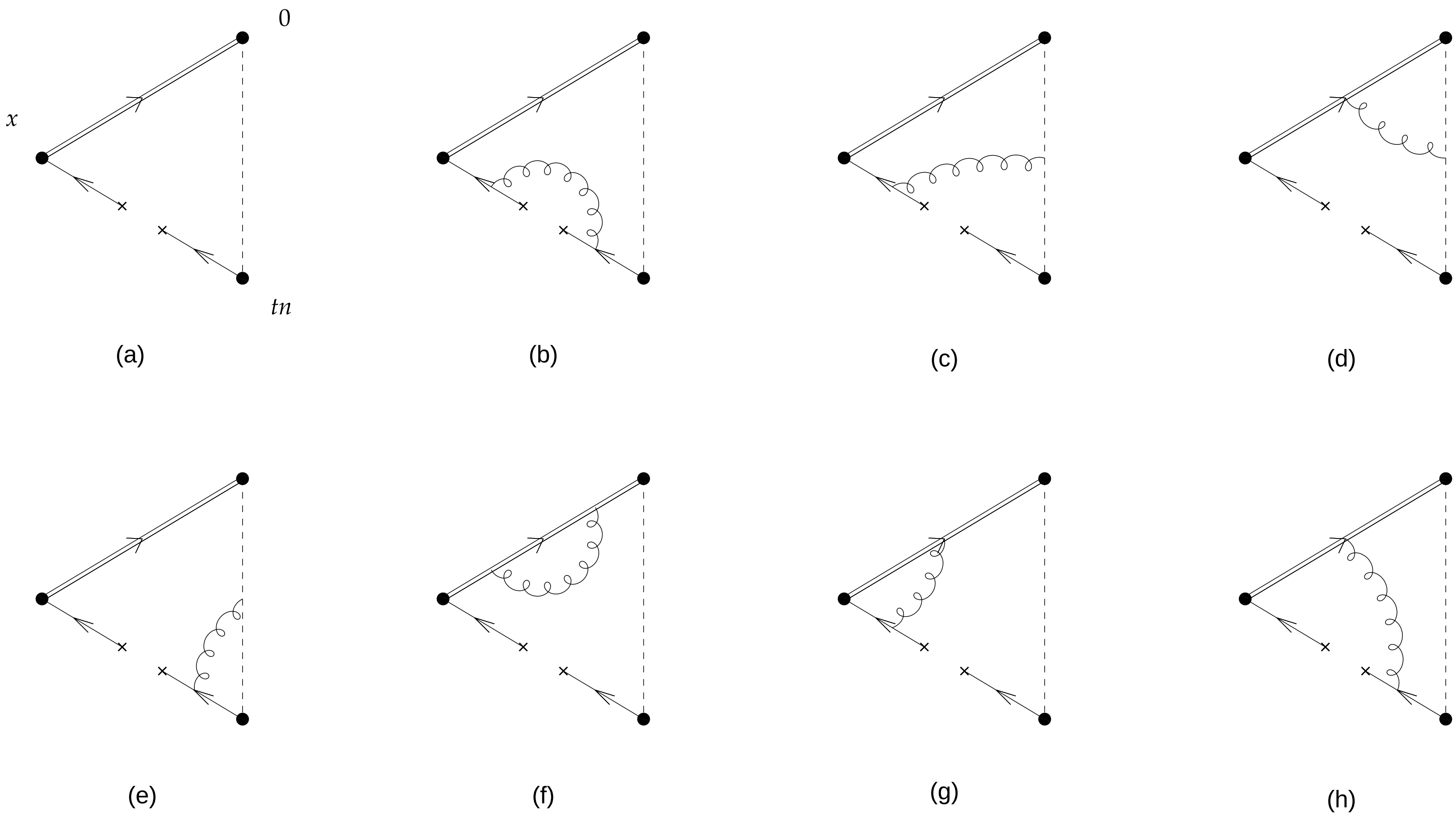
$O_+(t) = \bar{q}(tn)\,\not{n}\,[tn,0]\Gamma\, h_v(0),$

$k < 0$- external momentum variable, $\{...\}$ - a trace

correlator: the perturbative diagrams



correlator: the condensate diagrams



• the result for diagrams: schematically

$$T^{QCD \oplus HQET}(t,k) = \int_0^\infty \frac{dk'}{k'-k-i\epsilon} \int_0^\infty d\omega e^{-i\omega t} \rho(k',\omega,\mu)$$

• The hadronic dispersion relation: B-meson $\oplus$ {excited B-states}

$$T(t,k) = \frac{1}{2} F^2(\mu) \frac{1}{\bar{\Lambda} - k - i\epsilon} \int_0^\infty d\omega \, e^{-i\omega t} \phi_+^B(\omega,\mu) + \ldots$$

.

$F(\mu)$ -B decay constant in HQET

• duality, ω_0 threshold, Borel transform., 2pt SR for $F(\mu)$

$$\Rightarrow \lim_{\omega \to 0} \phi(\omega) \sim \omega, \quad \lim_{\omega \to \infty} \phi(\omega) \sim -\log(\omega/\mu)/\omega$$

- the sum rule fitted to an explicit ansatz for $\varphi_+^B(\omega)$,

$$\phi_+^B(\omega,\mu = 1\ \text{GeV}) = \frac{4\lambda_B^{-1}}{\pi}\frac{\omega}{\omega^2+1}\left[\frac{1}{\omega^2+1} - \frac{2(\sigma_B-1)}{\pi^2}\ln\omega\right],$$

(ω in units of GeV) $\quad \lambda_B^{-1} = \int_0^\infty \phi_+^B(\omega,\mu)/\omega = (460 \pm 110\ \text{MeV})^{-1}$,

.

$$\sigma_B = \lambda_B \int_0^\infty \phi_+^B(\omega,\mu)\log(\mu/\omega)/\omega = 1.4 \pm 0.4$$

$\mu = 1$ GeV

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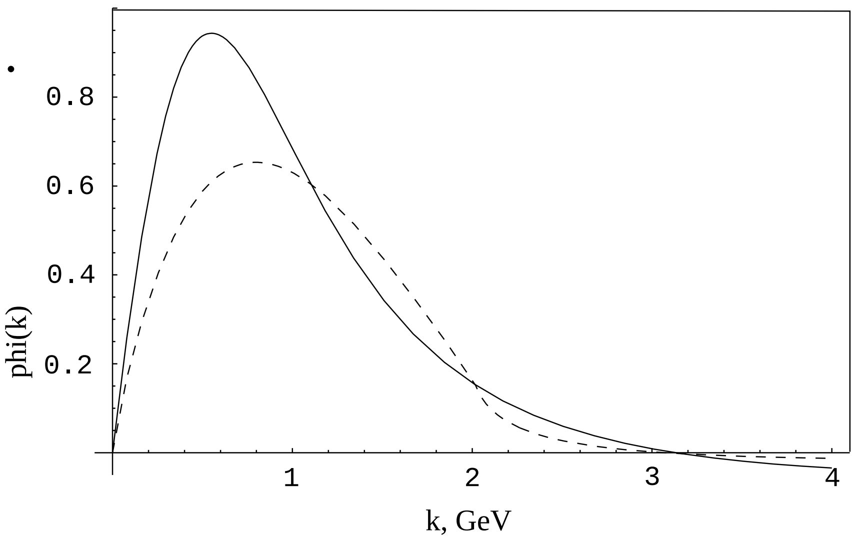


Model for $\varphi_+^B(k)$ (solid)
perturbative sum rule (dashed)
M=0.45 GeV, $\omega_0 = 1$ GeV (dashed)

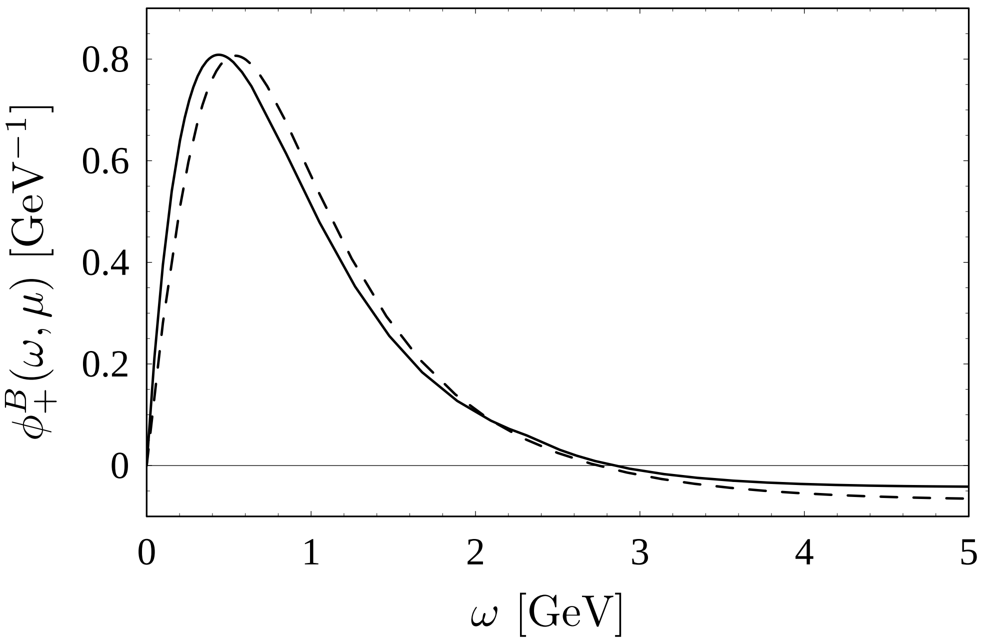
.

[V. M. Braun et al. hep-ph/0309330]

- A hybrid model for $\phi(\omega, \mu)$ [Lee, Neubert]:

inputs: Grozin-Neubert exponential model at small ω, "glued" to the radiative tail at large ω :

agrees with Braun-Ivanov-Korchemsky model at certain large Λ_{UV}



solid (dashed) is the hybrid (QCD SR) model

Applying LCSR to $B \to \gamma l \nu$

- Matching the LCSR for $B \to \gamma l\nu$ amplitude
to the factorization formula with $\int d\omega \phi_+^B(\omega)/\omega$

- the estimate $\lambda_B = 600$ MeV
[P. Ball and E. Kou, (2003)]

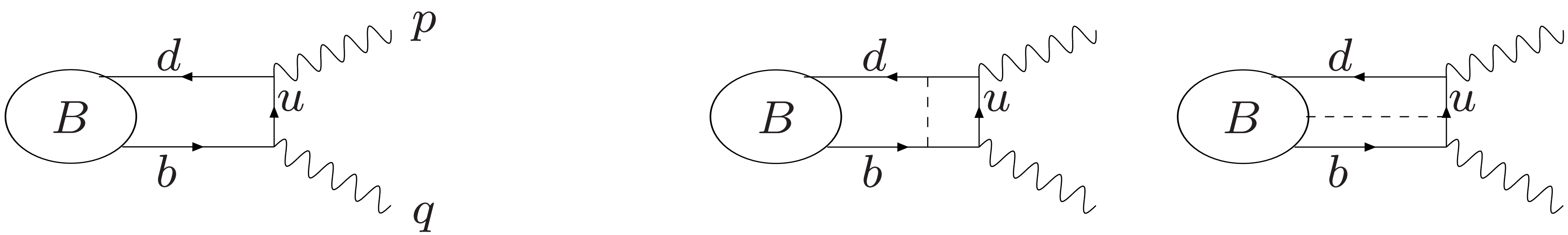
- the $O(1/m_b)$ long-distance photon emission (photon DA's) in $B \to \gamma l\nu$ is numerically large !
[see also A.K., G.Stoll, D.Wyler , (1995)]

LCSR: relating λ_B to $f_{B\to\pi}(0)$

. [A.K., T. Mannel, N.Offen PLB(2005), hep-ph/0504091]

- The correlator:

$$F^{(B)}_{\mu\nu}(p,q) = i\int d^4x\, e^{ip\cdot x}\langle 0|T\left\{\bar{d}(x)\gamma_\mu\gamma_5 u(x), \bar{u}(0)\gamma_\nu b(0)\right\}|\bar{B}^0(p+q)\rangle\,.$$



$q^2 = 0,\ p^2 < 0,\ |p^2| \gg \Lambda^2_{QCD},$ u-quark propagates near LC .

The sum rule

- OPE result, the LO diagram: only $\phi_-^B(u)$ contributes

$$F_{\mu\nu}^{(B)} = 2if_B \int\limits_0^\infty \frac{d\omega}{m_B\omega - p^2}\phi_-^B(\omega)p_\mu p_\nu + \dots ,$$

- Hadronic dispersion relation:

$$\langle \pi(p)|\bar{u}\gamma_\mu b|B(p+q)\rangle = f^+_{B\pi}(q^2)(2p_\mu + q_\mu) + \dots$$

$$F_{\mu\nu}^{(B)} = \left\{ \frac{2if_\pi f^+_{B\pi}(0)}{-p^2} + \int\limits_{s_h}^\infty ds \frac{\rho^h(s)}{s-p^2} \right\} p_\mu p_\nu + \dots ,$$

apply duality in pion channel⊕ Borel transformation.

see also [F. De Fazio, T. Feldmann and T. Hurth; arXiv:hep-ph/0504088]

- The relation: (using $s_\pi^0 \ll m_B^2$):

$$\frac{1}{\lambda_B} = \frac{f_\pi f_{B\pi}^+(0) m_B}{f_B M^2 (1 - e^{s_0^\pi / M^2})} \, .$$

- inputs: LCSR for $B \to \pi$ form factor (in terms of pion DA's), 2pt sum rule for f_B

- 3-particle B meson DA's, enter
1) soft-gluon diagram
2) indirectly, violation of WW relation
   estimated - a few %

- the result: $\lambda_B = 440 \pm 100$ MeV

- future: $O(\alpha_s) \oplus$ renormalization

Summary on the inverse moment

$1/\lambda_B = \int_0^\infty d\omega \frac{\phi_+^B(\omega)}{\omega}$ renorm. scale $\sim$ 1 GeV

Method	λ_B [MeV]	Ref.
2pt SR in HQET,LO	$\simeq 350$	Grozin,Neubert
2pt SR in HQET, NLO	440 ± 110	Braun, Ivanov,Korchemsky
LCSR for $B \to \gamma l \nu_l$	$\simeq 600$	Ball, Kou
"inverted" LCSR for $B \to \pi$	460 ± 160	A.K.,Mannel, Offen
first moments +Ansatz	480 ± 55	Lee, Neubert

Summary

$*$ B-meson DA: ϕ_+^B an important element of factorization in exclusive B decays; ϕ_-^B determines the "soft" $B \to \pi$ form factor
$*$ what is the role of 3-particle DA's in phenomenology?

$*$ QCD sum rules (CZ type, LCSR) combined with model-independent relations agree on λ_B, start getting σ_B and the shape of ϕ_+^B,
can we make uncertainties smaller ?

$*$ is it possible to estimate λ_B and other parameters on the lattice?

$*$ consistent definitions of DA's in HQET (SCET)
what is ϕ_+^B at finite m_b ?, is there a radiative tail?