$m_{\rm b}$ and $f_{\rm B_s}$ from a combination of HQET and QCD with unphysically light b-quarks

Damiano Guazzini

DESY Zeuthen
Platanenallee 6, 15735 Zeuthen Germany

in collaboration with Rainer Sommer (DESY-Zeuthen)

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Heavy-light meson physics on the lattice (quenched approx.)

To investigate the heavy-light meson properties one has to deal with two different scales:

- ⊙ The inverse of the wavelenght of the light quark $\sim \Lambda_{\rm QCD}$
  Light quark $\Rightarrow$ widely spread object $\Rightarrow$ large lattice
- ⊙ Heavy quark mass (e.g. $m_{\rm b} \sim 5{\rm GeV}$)
  Heavy quark $\Rightarrow$ highly localized $\Rightarrow$ high lattice resolution, $O(100^4)$

$\Rightarrow$ Unfeasible in naive lattice QCD $\Longrightarrow$ Need of an alternative approach:
Step scaling method
Mild extrapolation to the B region

Recourse to an effective theory $\Longrightarrow$ Heavy Quark Effective Theory (HQET)
$\Longleftrightarrow$ Accurate expansion in $\Lambda_{\rm QCD}/m_{\rm Q}$

$$\mathcal{L}_{\rm HQET} = \bar\psi_{\rm h} D_0 \psi_{\rm h} - \frac{c_{\rm kin}}{2m_{\rm Q}} \bar\psi_{\rm h} \mathbf{D}^2 \psi_{\rm h} - \frac{c_{\rm spin}}{2m_{\rm Q}} \bar\psi_{\rm h} \mathbf{B} \cdot \sigma \psi_{\rm h} + \ldots$$

- ◇ two component field $\psi_{\rm h}$, static quarks propagate only forward in time
- ◇ spin-flavour symmetry
- ◇ symmetry broken in $\mathcal{L}_{\rm HQET}$ by the $O(1/m_{\rm Q})$ terms

Step scaling method (1)

The method has been designed to deal with two scale problems in lattice QCD using the relativistic QCD Lagrangian. [M. Guagnelli et al., Phys. Lett. B **546** (2002) 237]

Let us have a physical observable $O(E_l, E_h)$ with $E_l \ll E_h$

Main assumption: The finite size effects affecting O have a mild dependence upon E_h and can be treated by a smooth extrapolation

Defining:

$$\sigma_O(E_l, E_h, 2L) = \frac{O(E_l, E_h, 2L)}{O(E_l, E_h, L)}$$

A total decoupling would be: $\sigma_O(E_l, E_h, 2L) \simeq \sigma_O(E_l, 2L)$.
In practice E_h never decouples, but one can parametrize the residual dependence

$$\sigma_O(E_l, E_h, 2L) = \sigma_O(E_l, 2L) + \frac{\alpha^{(1)}(E_l, 2L)}{E_h} + \frac{\alpha^{(2)}(E_l, 2L)}{E_h^2} + \cdots$$

Step scaling method (2)

$\Longrightarrow$ Extrapolation to regions of physical interest where direct simulations are unfeasible

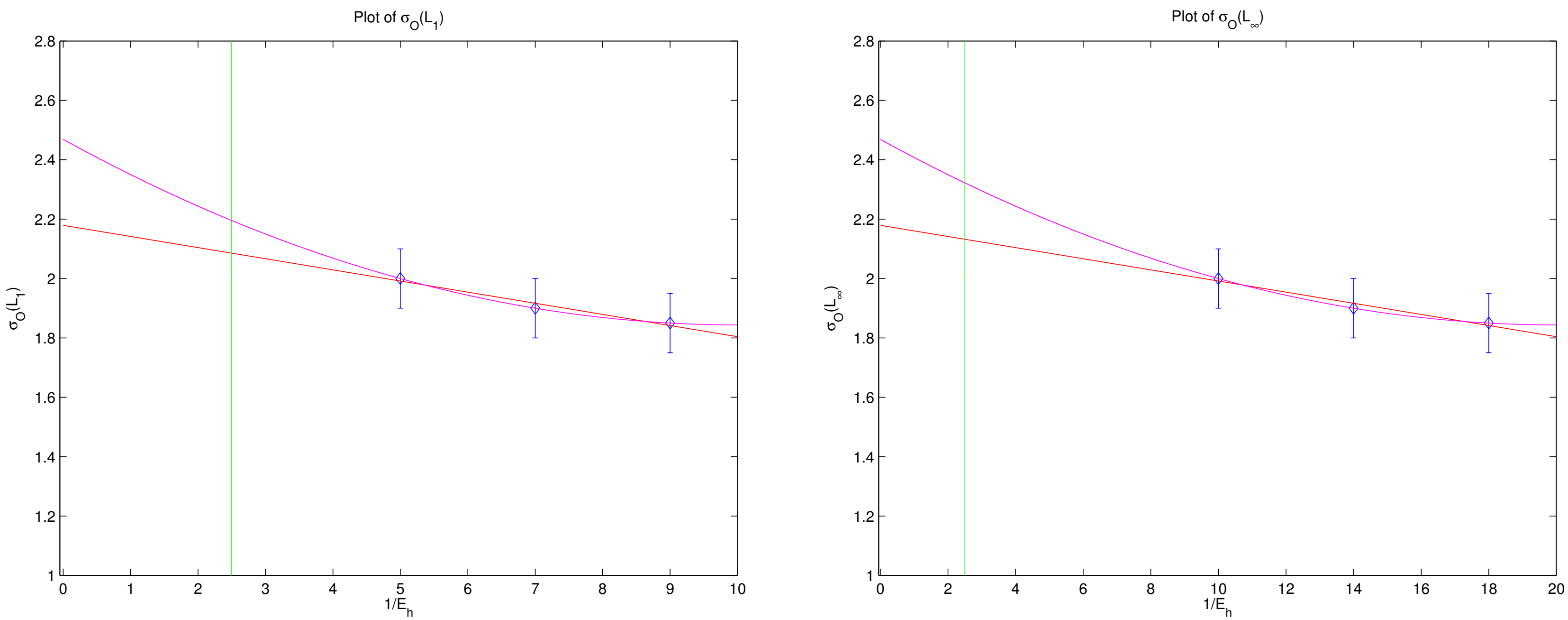
The steps:

$$O(E_l, E_h, L_\infty) = [O(E_l, E_h, L_0)] \cdot [\frac{O(E_l, E_h, 2L_0)}{O(E_l, E_h, L_0)}] \cdot [\frac{O(E_l, E_h, L_\infty)}{O(E_l, E_h, 2L_0)}]$$

$$= [O(E_l, E_h, L_0)] \cdot \sigma_O(E_l, E_h, 2L_0) \cdot \sigma_O(E_l, E_h, L_\infty)$$

and each step can be extrapolated to the continuum limit.

But it still remains an EXTRAPOLATION. . .



Our strategy: STATIC+Step Scaling Method=INTERPOLATION

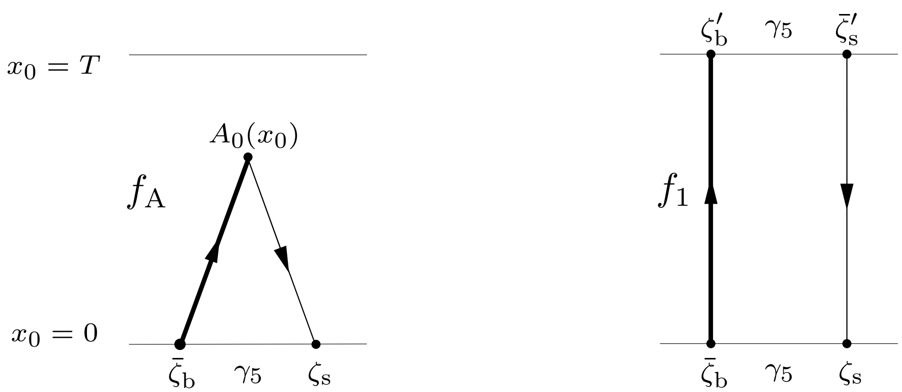
Correlation functions - QCD

Continuum: $A_0 = \bar{\psi}_s \gamma_0 \gamma_5 \psi_b$ $\quad f_{B_s}\sqrt{M_{B_s}} = Z_A \langle 0|A_0|B_s\rangle$

On the lattice one computes the correlation function f_A and the boundary-to-boundary correlator f_1:

$$f_A(x_0) = -\frac{a^6}{2}\sum_{\mathbf{y},\mathbf{z}} \langle A_0(x)\bar{\zeta}_b(\mathbf{y})\gamma_5\zeta_s(\mathbf{z})\rangle$$

$$f_1 = -\frac{a^{12}}{2L^6}\sum_{\mathbf{y},\mathbf{z},\mathbf{u},\mathbf{w}} \langle \bar{\zeta}'_s(\mathbf{u})\gamma_5\zeta'_b(\mathbf{w})\bar{\zeta}_b(\mathbf{y})\gamma_5\zeta_s(\mathbf{z})\rangle$$



and the meson decay constant is defined as $\quad f_{B_s}(L) = \frac{2}{\sqrt{L^3 M_{B_s}(T/2)}}\frac{f_A^R(T/2)}{\sqrt{f_1}} \overset{T,L\to\infty}{=} f_{B_s}$

for the meson mass we define $\quad \Gamma(L,M) = \frac{1}{2a}[\ln(f_A(x_0-a))/\ln(f_A(x_0+a))] \overset{L\to\infty}{\longrightarrow} m_B$

Pseudoscalar meson mass - HQET

Expansion in terms of the inverse of the mass of the heavy quark

The axial current reads $A_0 = \bar{\psi}_{\rm s}\gamma_0\gamma_5\psi_{\rm b} \longrightarrow A_0^{\rm stat} = \bar{\psi}_{\rm s}\gamma_0\gamma_5\psi_{\rm h}$

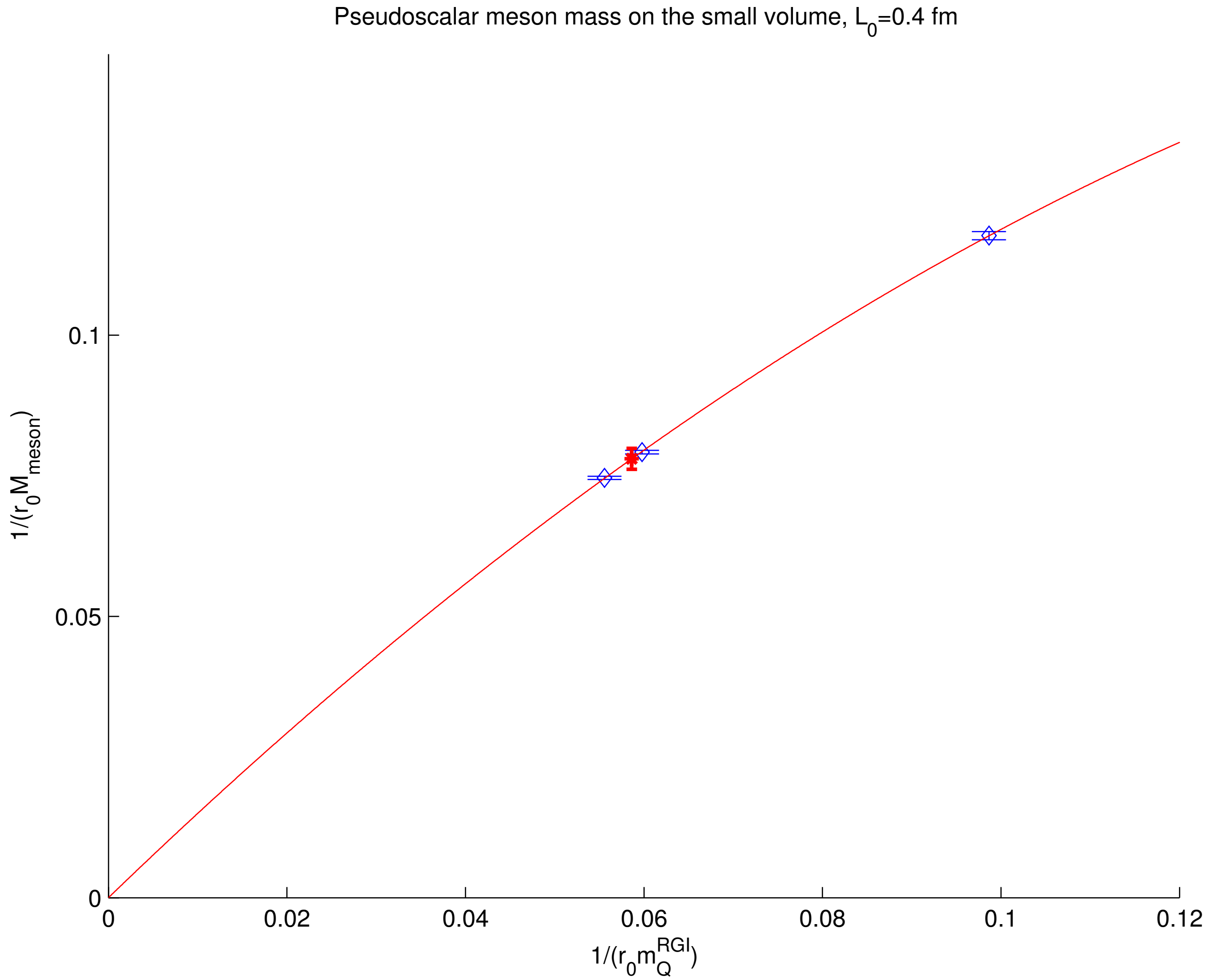
we define $\sigma_{\rm m,stat}(\bar{g}^2(L)) \equiv 2L[\Gamma_{\rm stat}(2L,M) - \Gamma_{\rm stat}(L,M)]$

One can then expand the pseudoscalar meson mass and the finite-size scaling

$$
\begin{aligned}
M_{\rm PS}(L) &= m_{\rm Q} + \Gamma^{\rm R}_{\rm stat}(L) + O(1/m_{\rm Q}) \\
\Sigma_P(L) &= \frac{M_{\rm PS}(2L)}{M_{\rm PS}(L)} = \frac{m_{\rm Q} + \Gamma^{\rm R}_{\rm stat}(2L) + O(1/m_{\rm Q})}{m_{\rm Q} + \Gamma^{\rm R}_{\rm stat}(L) + O(1/m_{\rm Q})} \\
&= 1 + \frac{\Gamma^{\rm R}_{\rm stat}(2L) - \Gamma^{\rm R}_{\rm stat}(L)}{m_{\rm Q}} + O(1/m_{\rm Q}^2) \\
&= 1 + \frac{2L\,[\Gamma_{\rm stat}(2L) - \Gamma_{\rm stat}(L)]}{2Lm_{\rm Q}} + O(1/m_{\rm Q}^2) \\
&= 1 + \frac{\sigma_{\rm m,stat}}{2Lm_{\rm Q}} + O(1/m_{\rm Q}^2) \\
&= 1 + \frac{\sigma_{\rm m,stat}}{2L\left[m_{\rm Q} + \Gamma^{\rm R}_{\rm stat}(2L)\right]} + O(1/M_{\rm PS}^2(2L)) \\
&= 1 + \frac{\sigma_{\rm m,stat}}{2LM_{\rm PS}(2L)} + O(1/M_{\rm PS}^2(2L))
\end{aligned}
$$

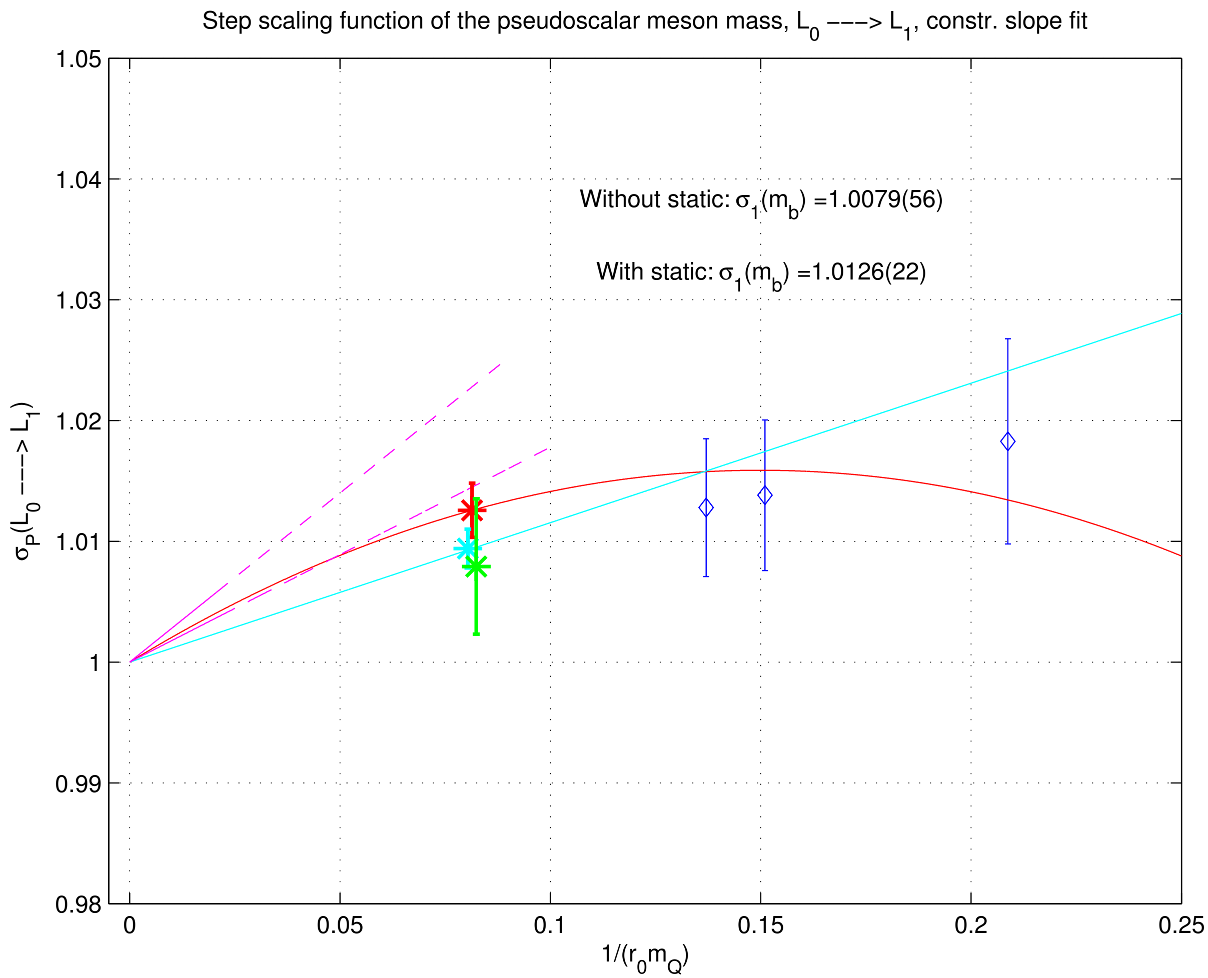
(allows to avoid a pert. definition of the quark mass)

Pseudoscalar meson mass - Small volume



fitting curve: $$\frac{1}{r_0 M_{\mathrm{meson}}} = b \cdot \left(\frac{1}{r_0 m_{\mathrm{Q}}^{\mathrm{RGI}}}\right) + c \cdot \left(\frac{1}{r_0 m_{\mathrm{Q}}^{\mathrm{RGI}}}\right)^2$$

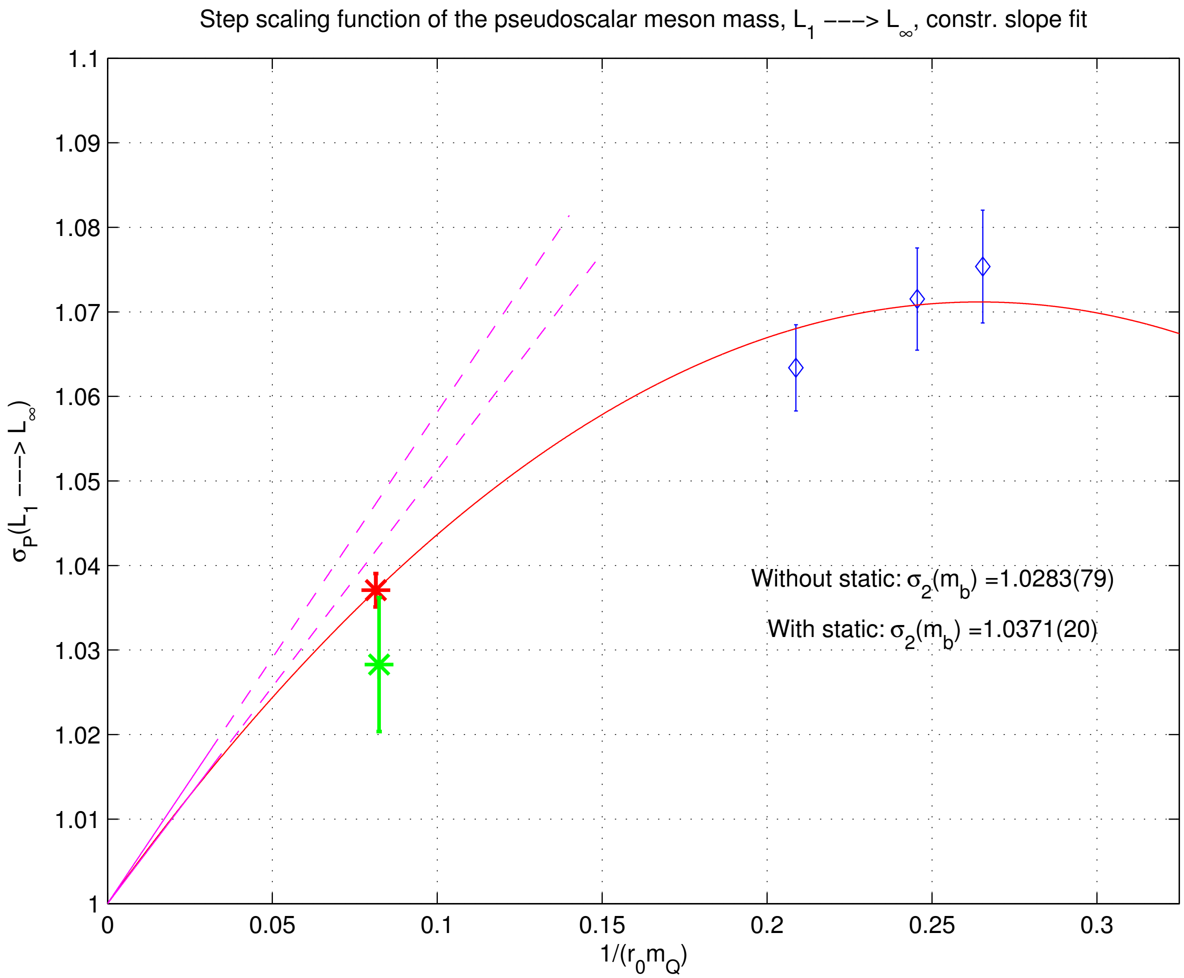
σ_1 Pseudoscalar meson mass - Static + Step scaling



$$\sigma_1(m_\mathrm{b}) = 1.0079 \pm 0.0056, \quad \text{without the static constraint}$$

$$\sigma_1(m_\mathrm{b}) = 1.0126 \pm 0.0022, \quad \text{with the static constraint}$$

σ_2 Pseudoscalar meson mass - Static + Step scaling



$$\sigma_2(m_\mathrm{b}) \quad = \quad 1.0283 \pm 0.0079, \qquad \text{without the static constraint}$$

$$\sigma_2(m_\mathrm{b}) \quad = \quad 1.0371 \pm 0.0020, \qquad \text{with the static constraint}$$

Computation of the b-quark mass

$$M_{\rm PS} = M_{\rm PS}(L_0) \cdot \sigma_1(L_1) \cdot \sigma_2(L_\infty)$$

- ⊙ Iteration of the procedure for several b-quark masses in the range: $6.5\ {\rm GeV} \leq m_{\rm b}^{\rm RGI} \leq 7.0\ {\rm GeV}$
- ⊙ Matching the computed $M_{\rm PS}$ with the experimental one

⇒ Extraction of the b-quark mass

Preliminary quenched results:

Only RomeII	$m_{\rm b}^{\rm RGI} = 6.932(83)(69)$ GeV
Static+RomeII	$m_{\rm b}^{\rm RGI} = 6.823(33)(68)$ GeV

Which can be compared with

$m_{\rm b}^{\rm RGI} = 6.685(99)(46)$ GeV, HYP2
$m_{\rm b}^{\rm RGI} = 6.671(99)(46)$ GeV, HYP1

[M. Della Morte et al., see N. Garron's talk]

Decay constant - HQET

We define:

$$X_1(g_0, L/a) = \frac{f_{\rm A}^{\rm stat}(T/2)}{\sqrt{f_1^{\rm stat}}}$$

We are interested in renormalized and improved quantities:

$$X_{\rm RGI}(L) = \lim_{a/L\to 0} -Z_{\rm RGI}(g_0) X_1(g_0, L/a) = \frac{-f_{\rm B}(L)\sqrt{m_{\rm B}(L)}\, L^{3/2}}{2C_{\rm PS}(M/\Lambda_{\rm QCD})} + O(1/m_{\rm Q})$$

where $C_{\rm PS}$ comes from continuum perturbation theory; uncertainty is $O(\alpha_{\rm s}^3(M_{\rm b}))$
$\gamma^{\rm PS}$ from [K. G. Chetyrkin and A. G. Grozin, Nucl. Phys. B **666** (2003) 289]

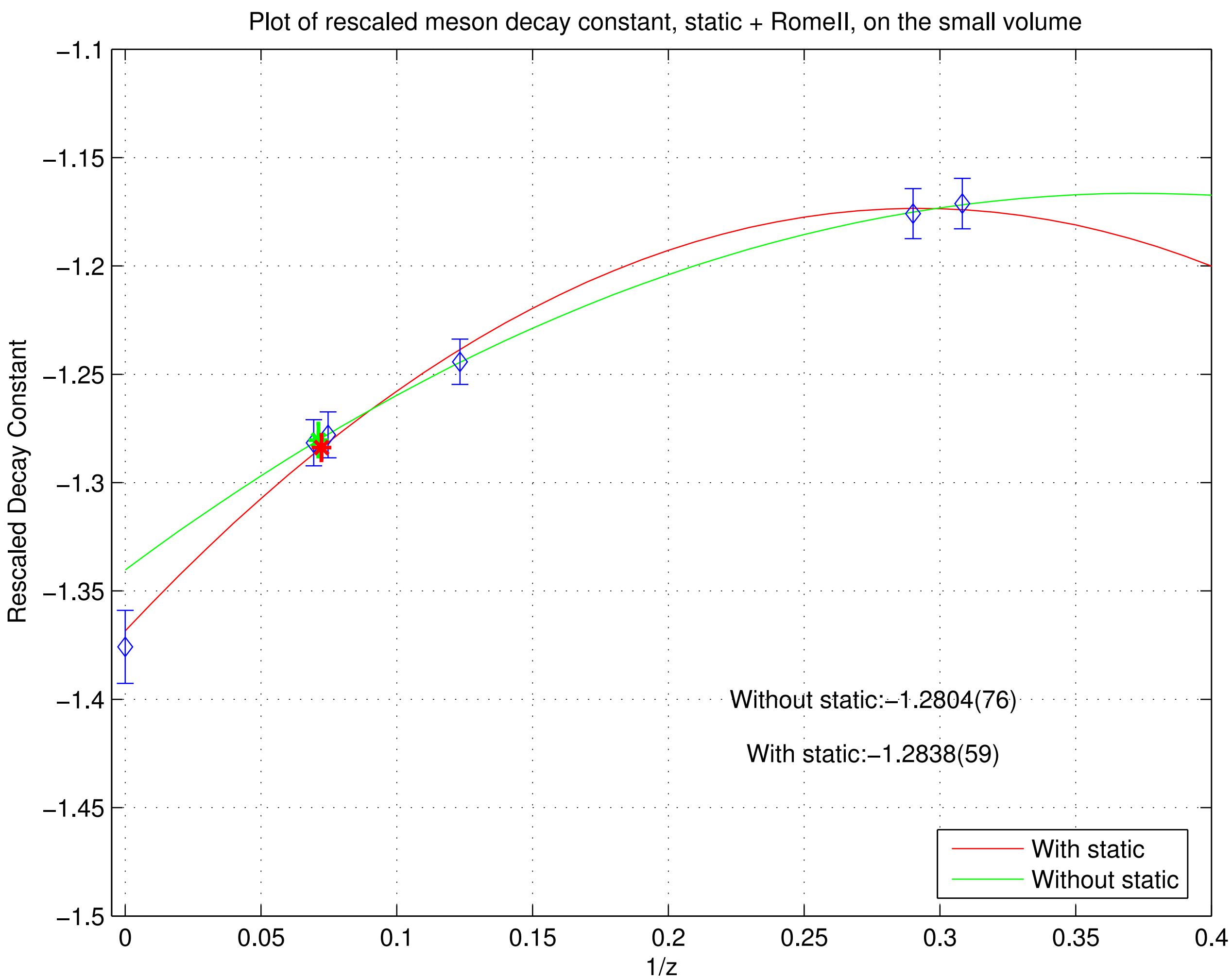
$$(A_0^{\rm stat})_{\rm RGI} = Z_{\rm RGI}(g_0)\cdot A_0^{\rm stat}, \qquad Z_{\rm RGI}(g_0) = \frac{\Phi_{\rm RGI}}{\Phi(\mu = 1/L)}\cdot Z_{\rm A}^{\rm stat}(g_0, L/a)$$

where $Z_{\rm A}^{\rm stat}(g_0, L/a)$ NP computed

$$\frac{\phi_{\rm RGI}}{\phi(\mu)} = [2b_0\bar{g}^2(\mu)]^{-\gamma_0/2b_0} \exp\left\{-\int_0^{\bar{g}(\mu)} {\rm d}g\left[\frac{\gamma(g)}{\beta(g)} - \frac{\gamma_0}{b_0 g}\right]\right\}, \quad \text{NP known [ALPHA]}$$

$$X_{\rm RGI}(L) = \frac{\Phi_{\rm RGI}}{\Phi(1/L)}\cdot \lim_{a/L\to 0} X_{\rm SF}(g_0, L/a) = O(E_l, E_h, L)$$

Decay constant - Small volume

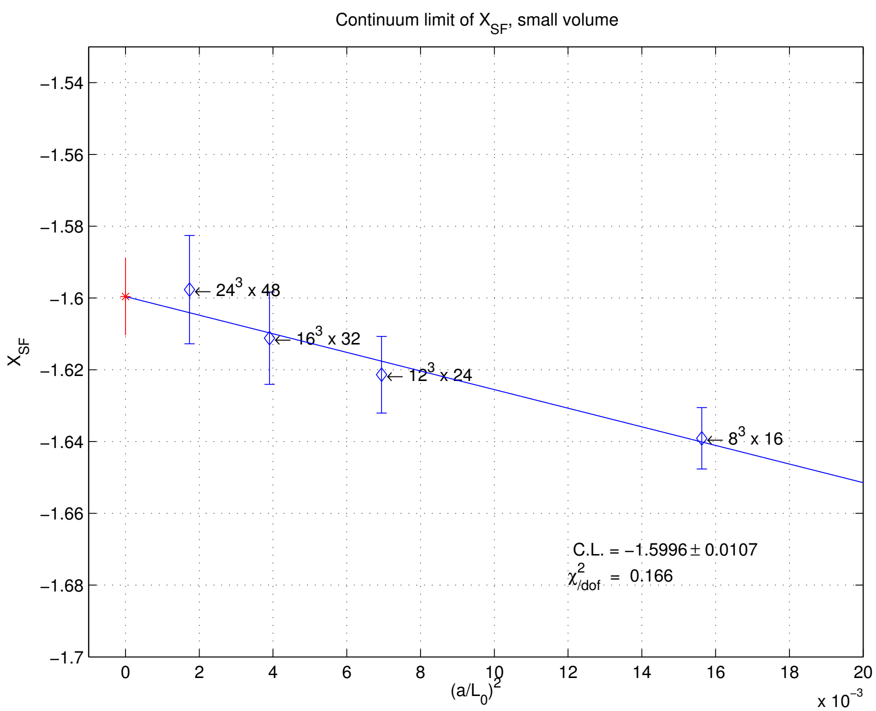


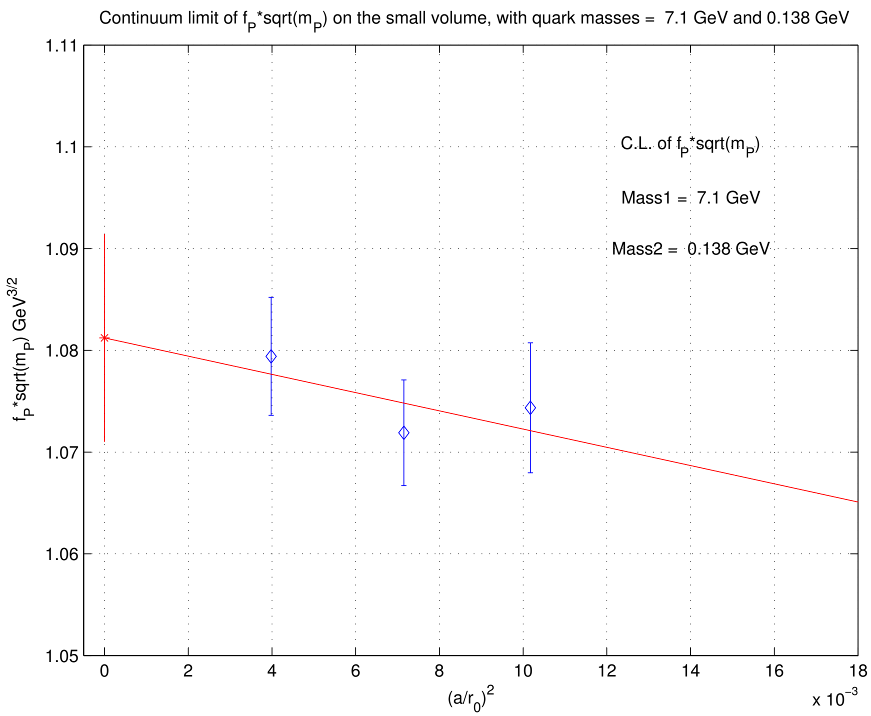
$X^{\rm b}_{\rm RGI}(L_0) = -1.2804(76)$ without the static point
$X^{\rm b}_{\rm RGI}(L_0) = -1.2838(59)$ static point included

$O(L_\infty) = O(L_0)\sigma(2L_0)\sigma(L_\infty)$

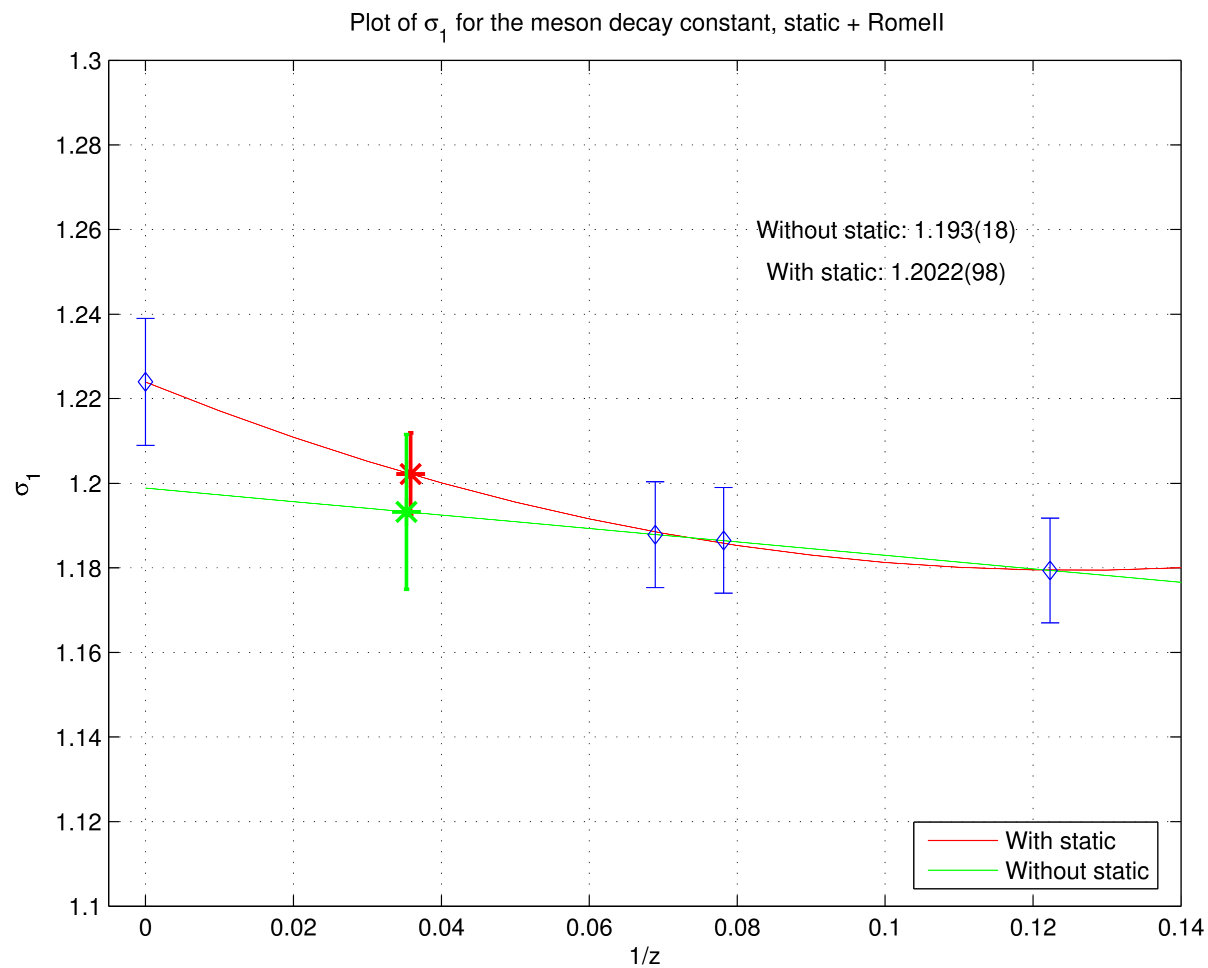
$1/z = 1/(L_0 M)$

Some continuum limits - Small volume





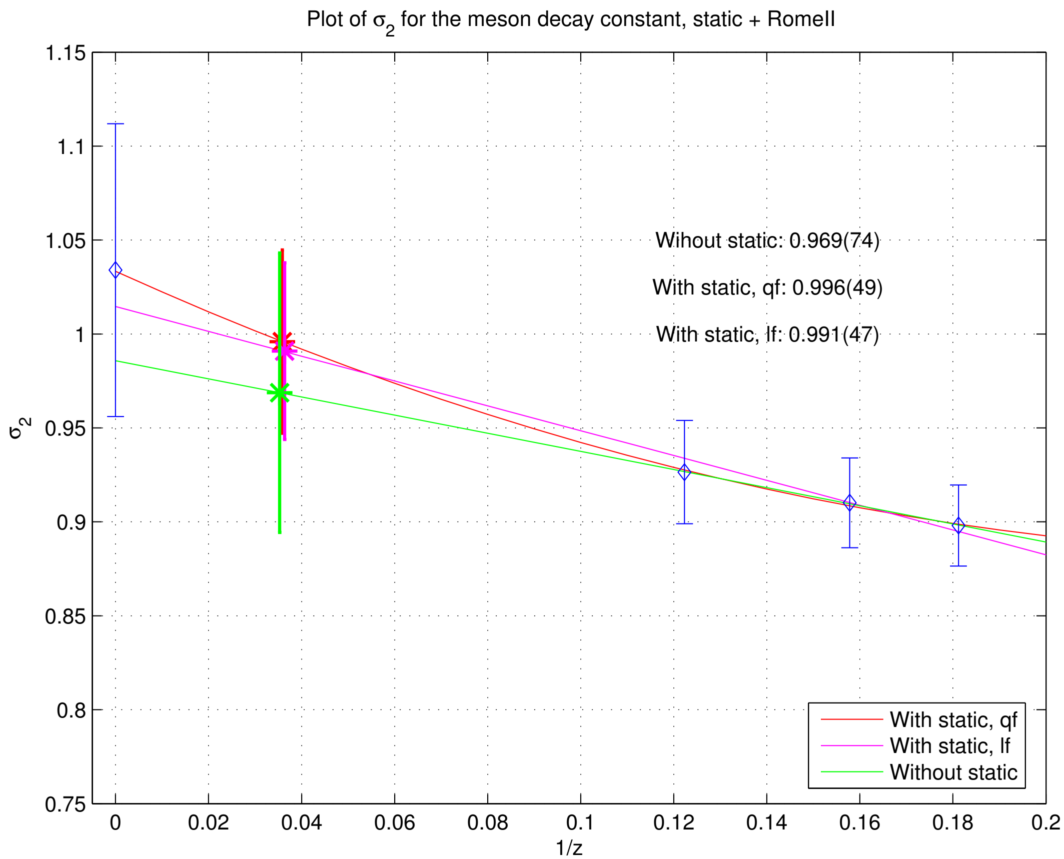
σ_1 Decay constant: Static + Step scaling



$X^{\mathrm{b}}_{\mathrm{RGI}}(L_0) = -1.193(18)$ without the static point
$X^{\mathrm{b}}_{\mathrm{RGI}}(L_0) = -1.2022(98)$ static point included

$$O(L_\infty) = O(L_0)\sigma(2L_0)\sigma(L_\infty)$$
$$1/z = 1/(L_1 M)$$

σ_2 Decay constant: Static + Step scaling



$\sigma_2^{\rm b} = 0.969 \pm 0.074$ without static

$\sigma_2^{\rm b} = 0.996 \pm 0.049$ static incl., qf

$\sigma_2^{\rm b} = 0.991 \pm 0.047$ static incl., lf

$r_0^{3/2} X_{\rm RGI}(L_\infty) = 1.78(13)$ [ALPHA coll.]

$O(L_\infty) = O(L_0)\sigma(2L_0)\sigma(L_\infty)$

$1/z = 1/(L_1 M)$

Deacy constant - Results

For the $\mathrm{B_s}$ meson decay constant we obtain the preliminary quenched result

Only RomeII	$f_{\mathrm{B_s}} = 188(15)(4)$ MeV
Static+RomeII	$f_{\mathrm{B_s}} = 194(9)(4)$ MeV

Which can be compared with

$$f_{\mathrm{B_s}} = 205(12)(10) \text{ MeV}$$

[M. Della Morte et al., Phys. Lett. B **581** (2004) 93]

Summary

- We compute on the lattice the bottom-strange decay constant and pseudoscalar meson mass in the static limit of HQET
- We combine them with the results of the *Tor Vergata* group obtained using the relativistic QCD Lagrangian and the SSM to interpolate to the B-physics observables
- The comparisons of QCD observables with the predictions of HQET represents a successful test of the effective theory
- The validity of the SSM introduced by RomeII has been succesfully tested
- The HQET results allowed a considerable improvement in the determination of the fitting curves which now interpolate B-physics observables with a higher precision
- Preliminary quenched results:

$$m_{\rm b}^{\rm RGI} = 6.823(33)(68)\ {\rm GeV} \quad \Rightarrow \quad m_{\rm b}(m_{\rm b}) = 4.384(21)(44),\ \overline{MS}\ \text{scheme}$$

$$f_{\rm B_s} = 194(9)(4)\ {\rm MeV}$$

Outlook

- Extension of the method to a precise computation of other B-physics observables (e.g. $B - \bar{B}$ mixing amplitude)

- Inclusion of the $O(1/m_{\mathrm{Q}})$ corrections in HQET (non-perturbatively)

- Dynamical fermions