Light-Cone Sum Rules in Soft-Collinear Effective Theory

Thorsten Feldmann

(Siegen University)

– Mini-Workshop on “Flavour Physics” –
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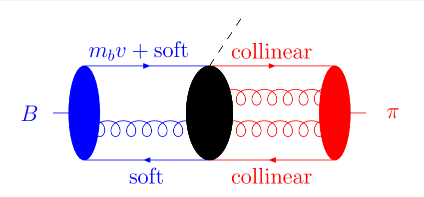
based on work with F. De Fazio and T. Hurth, hep-ph/0504088
– to appear in Nucl.Phys.B –

Outline

Momentum regions in $B \to \pi$ transitions (large recoil)

$\longrightarrow$ method of regions [Beneke/Smirnov]



$E_\pi = \mathcal{O}(m_b)$
$\Lambda = \mathcal{O}(\Lambda_{QCD})$

- long-distance modes:
  - HQET fields: $\Delta p \sim \Lambda$
  - soft spectators: $p_s^\mu \sim \Lambda$
  - collinear quarks and gluons: $E_c \sim m_b, \quad p_c^2 \sim \Lambda^2$
- short-distance modes:
  - hard modes: $(\text{heavy} + \text{collinear})^2 \sim m_b^2$
  - hard-collinear modes: $(\text{soft} + \text{collinear})^2 \sim m_b \Lambda$

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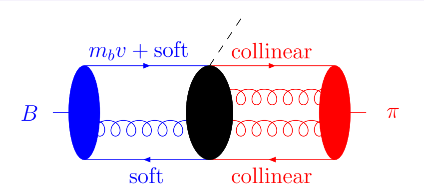

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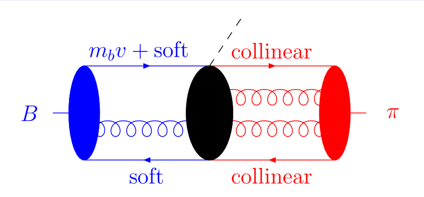

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Factorization: Why?

- Express form factors in terms of universal (process-independent) hadronic quantities!
- Determine asymptotic dependence on heavy-quark mass! (power counting, Sudakov logs, . . .)
- Renormalization-group improved perturbation theory:
  - Compare: $\alpha_s(m_b) \approx 0.2$ with $\alpha_s(\sqrt{m_b \Lambda}) \approx 0.3 - 0.4$
  - Parameter $a = \frac{\Gamma_0}{2\beta_0} \ln \frac{\alpha_s(\sqrt{m_b\Lambda})}{\alpha_s(m_b)} \approx 0.3 - 0.4$ not small!

  $\Rightarrow$ Resum large logarithms in Λ/m_b into short-distance coefficients.

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Effective theory construction

[Bauer/Fleming/Pirjol/Stewart, Beneke/TF et al., Chay/Kim, Neubert et al., . . . , 2001 – today]

- Separate short- and long-distance modes:

$$p_c^2,\, p_s^2 \ll p_{\rm hc}^2 \ll p_{\rm h}^2$$

$\rightarrow$ Use dimensional regularization: $d^4k \rightarrow \mu^{2\epsilon} d^{4-2\epsilon}k$

  - modes with $p^2 > \mu^2$ in short-distance coefficients
  - modes with $p^2 < \mu^2$ in matrix elements

- Two-step matching procedure:
  - integrate out hard modes at $\mu_1^2 \sim m_b^2$ $\longrightarrow$ $\mathrm{SCET_I}$
  - renormalization group in $\mathrm{SCET_I}$:
    $\longrightarrow$ evolution to $\mu_2^2 \sim m_b \Lambda_{\rm QCD}$
  - integrate out hard-collinear modes $\longrightarrow$ $\mathrm{SCET_{II}}$
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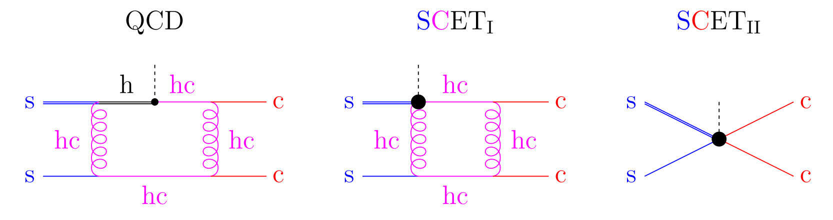
Example



- part of tree-level matching for QCD $\to$ SCET$_{\rm I}$
- part of one-loop matching for SCET$_{\rm I}$ $\to$ SCET$_{\rm II}$

Factorizable and Non-factorizable currents in $\mathrm{SCET_I}$

[Beneke/TF '03, see also Lange/Neubert '03]

- Factorization theorem:

$$F_i(q^2) \;=\; T_i^I(q^2)\,\xi_\pi(q^2) + \phi_+^B \otimes T_i^{II}(q^2) \otimes \phi_\pi + 1/m_b \text{ corrections}$$

- Non-factorizable current: $n_+ A_{\rm hc} = 0,\; n_- A_s = 0$

$$\langle \pi(p')|\bar\xi_{\rm hc}\, h_v|B(p)\rangle \equiv 2E_\pi\, \xi_\pi$$

Soft and collinear fields decouple in $\mathrm{SCET_{II}}$ Lagrangian,

but no regularization consistent with soft/collinear factorization.

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$$\langle \pi(p')|\bar\xi_{\rm hc}\, g A_{\rm hc}^\perp\, h_v|B(p)\rangle \equiv -\frac{m_B^2}{2}\,\frac{\alpha_s C_F}{4\pi}\,\Delta F_\pi$$

where

$$\Delta F_\pi = \frac{8\pi^2 f_B f_\pi}{3 m_B}\,\langle \omega^{-1}\rangle_+^B\,\langle \bar u^{-1}\rangle_\pi + \ldots$$

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Non-perturbative methods for $B \to \pi$ decays

Lattice QCD:

- Problem: cannot implement energetic pions!
- Way out: moving NRQCD ?

[Foley/Lepage et al., hep-lat/0209135,0509108]

(traditional) QCD light-cone sum rules:

- Separate hard and collinear dynamics
- Hard and hard-collinear scale not distinguished in OPE
- Heavy-quark limit, $m_b \to \infty$?

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Light-cone sum rules in SCET

[De Fazio/TF/Hurth '05]

Idea: First, integrate out hard modes: QCD $\to$ $\mathrm{SCET_I}$

- Consider correlation functions in $\mathrm{SCET_I}$, where exclusive final state (e.g. pion) is replaced by interpolating current.

  Separate soft and hard-collinear dynamics in correlator

- Dispersion relations between
  - (unphysical) region of large (hc) space-like momenta
  - physical spectral function, containing the hadronic state

Sum rule for non-factorizable matrix elements in $\mathrm{SCET_I}$ in terms of light-cone distribution amplitudes of B meson

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Correlation function for non-factorizable form factor

- Non-factorizable current in SCET_I : $J_0 = \bar{\xi}_{\rm hc}\, h_v$

- Consider interpolating axial-vector current in SCET_I :

$$J_\pi = -i\bar{\xi}_{\rm hc}\, \not{n}_+ \gamma_5\, \xi_{\rm hc} - i\left(\bar{\xi}_{\rm hc}\, \not{n}_+ \gamma_5\, q_s + \text{h.c.}\right)$$

  with $\langle 0 | J_\pi | \pi(p') \rangle = (n_+ p')\, f_\pi$

- Define correlation function:

$$\Pi_0(n_- p') = i \int d^4x\, e^{ip'x}\, \langle 0 | T\left[J_\pi(x) J_0(0)\right] | B(v) \rangle$$

  in Euclidean region, i.e.

$$(n_+ p') \simeq 2E_\pi = \mathcal{O}(m_b)\,, \qquad 0 > (n_- p') = \mathcal{O}(\Lambda)$$

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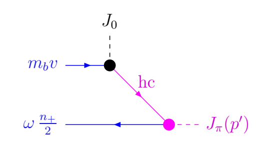
Calculation of the correlation function

- Tree level:

  Integrate out hard-collinear quark propagator



$$\Pi_0(n_-p') = f_B m_B \int_0^\infty d\omega\, \frac{\phi_-^B(\omega)}{\omega - n_-p' - i\eta}$$

- Result in terms of B-meson light-cone wave functions

$$\langle 0|\bar q_s^\beta(z)\, Y_s(z,0)\, h_v^\alpha(0)|B\rangle = -\frac{i f_B m_B}{4}\left[\frac{1+\not v}{2}\left\{2\tilde\phi_+^B(t) + \frac{\tilde\phi_-^B(t)-\tilde\phi_+^B(t)}{t}\,\not z\right\}\gamma_5\right]^{\alpha\beta}$$

- Radiative corrections:
  - hard-collinear effects perturbatively calculable
  - soft effects absorbed into B-meson LCWF(s)

Factorization of correlation function in SCET$_{\rm I}$

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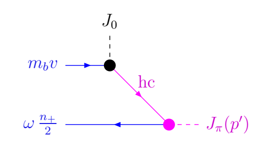

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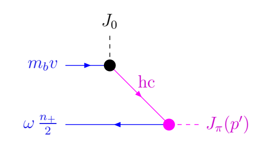

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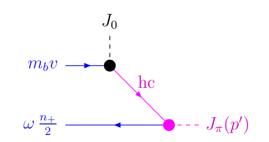

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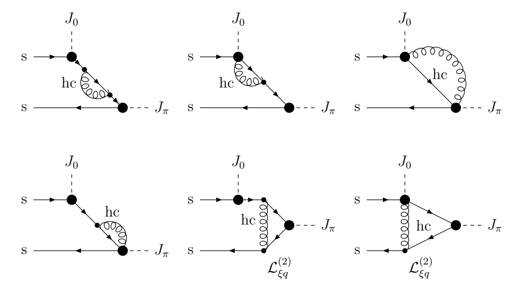
Hard-collinear loop diagrams at NLO

- Corrections involving $\phi_-^B(\omega)$:



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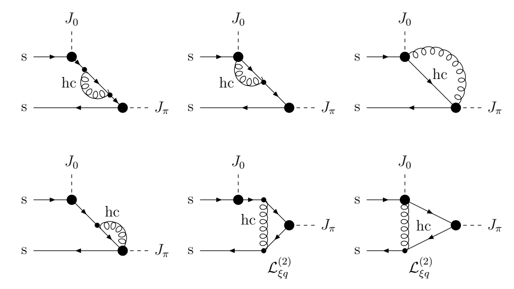

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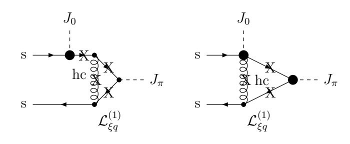

Renormalization-scale dependence

- Wilson coefficient in front of J_0 from matching/running; resums Sudakov logs between m_b and μ.
  [Bauer/Fleming/Pirjol/Stewart 2000]

- Scale-dependence induced by hard-collinear loops.
  (see above)

- RG evolution of soft LCWF:
  - Universal effect from *cusp anomalous dimension*.
  - Single logs only known for $\phi_+^B(\omega, \mu)$ [Lange/Neubert 2002]

Leading scale dependence $\sim \ln^2 \mu$ drops out $\surd$

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Dispersion relation / Continuum model

$$\Pi_0(n_- p') = \frac{1}{\pi} \int_0^\infty d\omega' \, \frac{\text{Im}\,[\Pi_0(\omega')]}{\omega' - n_- p' - i\eta}$$

Spectrum at tree level: $\text{Im}\,[\Pi_0(\omega')] = \int d\omega \, f_B m_B \, \phi_-^B(\omega) \, \pi\delta(\omega - \omega')$

- For $\omega' > \omega_s \approx m^2_{[3\pi]}/(n_+ p')$ "DUALITY"

  spectrum modelled by perturbative result in SCET_I:

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Borel transformation

- Result for ξ_π depends on:
  - threshold parameter ω_s
  - virtuality $(n_- p')$

- Reduce model dependence by Borel transform in $(n_- p')$:

$$\omega_M \, \hat{B}\left[\Pi_0^{\mathrm{res.}}\right](\omega_M, \omega_s) = \frac{1}{\pi} \int_0^{\omega_s} d\omega' \, e^{-\omega'/\omega_M} \, \mathrm{Im}\left[\Pi_0(\omega')\right] = (n_+ p') \, \xi_\pi \, f_\pi$$

Choose Borel parameter ω_M of order $\Lambda^2/(n_+ p')$:

- Continuum (large values of ω') exponentially suppressed
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Model dependence parametrized by ω_s and ω_M

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$$\xi_\pi(n_+p') = \frac{1}{f_\pi\,(n_+p')} \int_0^{\omega_s} d\omega'\, e^{-\omega'/\omega_M}\, \frac{1}{\pi}\mathrm{Im}\left[\Pi_0(\omega')\right]$$

$$= \frac{m_B\, f_B(\mu)}{(n_+p')\, f_\pi} \int_0^{\omega_s} d\omega\, e^{-\omega/\omega_M}\, \phi_-^B(\omega,\mu) + \mathcal{O}(\alpha_s)$$

- Further approximation: Expand $\phi(\omega)$ for small $\omega \leq \omega_s \ll \Lambda$, and use LO sum rule for f_π with *the same* parameters ω_M and ω_s

$$\Rightarrow \xi_\pi \approx \frac{4\pi^2 f_\pi f_B m_B}{(n_+p')^2}\, \phi_-^B(0)$$ [see also: Khodjamirian/Mannel/Offen '05]

Sensitive to $\phi_-^B(0) \simeq \langle\omega^{-1}\rangle_+^B = \int_0^\infty \frac{d\omega'}{\omega'}\, \phi_+^B(\omega')$ (WW relation)

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Numerical result

- Numerical estimate
  (including NLO radiative corrections, approx., excl. 3-particle LCDA)

$$\frac{C_i(\mu)}{C_i(m_b)} \cdot \xi_\pi(m_B,\mu) = 0.27 \pm 0.02\Big|_\mu \pm 0.07\Big|_{f_B\phi_-} \begin{smallmatrix}+0.05\\-0.08\end{smallmatrix}\Big|_{\rm s.r.} \begin{smallmatrix}+0.00\\-0.04\end{smallmatrix}\Big|_{\rm sys.}$$

- Dependence on Borel parameter (for $\omega_s = 200$ MeV, $\mu_0 = 1$ GeV)

(– – LO) (— NLO)

- Dependence on factorization scale (for $\omega_s = \omega_M = 200$ MeV)

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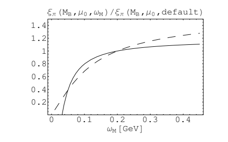
(– – LO)  (— NLO)

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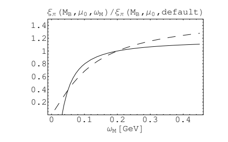
(– – LO)  (— NLO)

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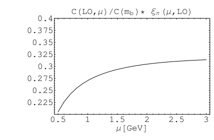
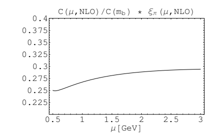
LO → 

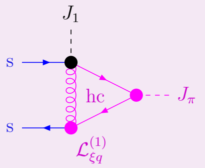
 ← NLO

Sum rule for factorizable current

- Factorizable current in SCET$_{\rm I}$: $J_1 = \bar\xi_{\rm hc}\, g \not{A}^{\perp}_{\rm hc}\, h_v$
- Define correlation function:

$$\Pi_1(n_- p') = i \int d^4x\, e^{ip'x}\, \langle 0 | T\,[J_\pi(x) J_1(0)]\, | B(v) \rangle$$

- Leading contribution at one loop:



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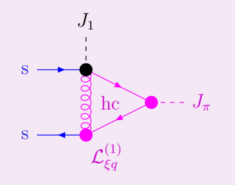

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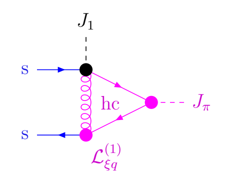

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Spectrum starts at order α_s:

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