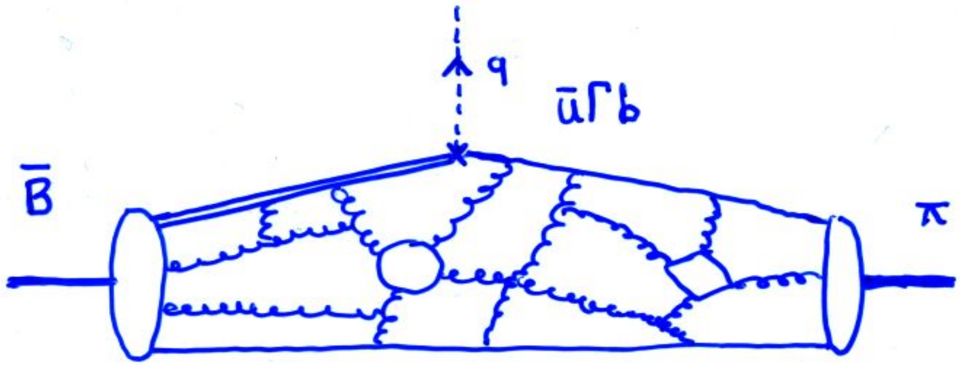


$$\langle \pi(p') | \bar{u}\gamma^\mu b | \bar{B}(p) \rangle =$$

$$f_+(q^2) \left(p^\mu + p'^\mu - \frac{M_B^2 - m_\pi^2}{q^2} q^\mu \right) + f_0(q^2) \frac{M_B^2 - m_\pi^2}{q^2} q^\mu$$

also tensor current $\rightarrow$ f_T

vector mesons $\rightarrow$ V, $A_{0,1,2}$, $T_{1,2,3}$

$q^2 \sim M_B^2$, E_π small : $f \sim M_B^{-1/2}$ (HQET)

$q^2 \sim 0$, $E_\pi \sim \frac{M_B}{2}$: $f \sim M_B^{-3/2}$

subject of this discussion

$\frac{d\Gamma(B \to \pi \ell \nu)}{dq^2}$ $\longrightarrow$

$|V_{ub} f|^2$

$Br(B \to \pi\pi)$ $\longrightarrow$

$\delta f \rightarrow \delta |V_{ub}|$

Want $\delta f < 10\%$

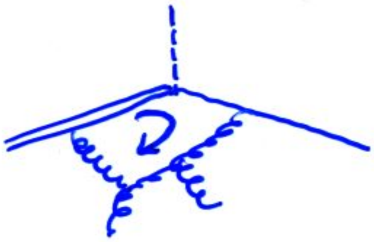
* What can be done in the heavy quark limit?

  - Status, open problems

* $f(0)$ mainly from QCD sum rules

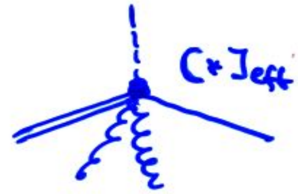
  - how reliable?

Heavy quark expansion



integrate out short-distance fluctuations →

Scale m_b



$\bar{u}\Gamma b$ $\qquad\qquad$ $\sum_i C^i * J^i_{eff}$

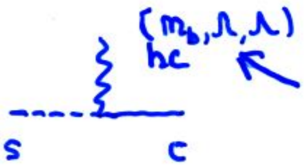
1) Relevant external momenta ($\bar{B}$ rest frame)

$\bar{B}$ consists of soft stuff $\qquad$ $p_b = mv + \ell$

$\ell_i \sim (\Lambda, \Lambda, \Lambda)$

π consists of collinear stuff $\qquad$ $k_i \sim (m_b, \Lambda, \frac{\Lambda^2}{m_b})$

$k = n_+ k \frac{n_-}{2} + k_\perp + n_- k \frac{n_+}{2}$



virtuality $m_b \Lambda$ - hard-collinear

Modes of $SCET_I$

2) Relevant operators

question of Λ/m_b power counting in SCET

$\bar{\xi} h_v$ $\qquad$ + $\qquad$ $\bar{\xi} A_\perp h_v$

at leading order in $1/m_b$

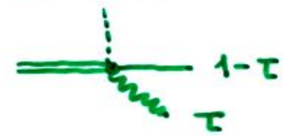
$$\bar u \Gamma b = C_\Gamma^{(A0)}(E)\,\bar\xi h_v + \frac{1}{m_b}\int d\tau\, C_\Gamma^{(B1)}(E,\tau)\int \frac{d(2Er)}{2\pi}\, e^{-2iEr\tau}\,\bar\xi A_{\perp}^{(hc)}(rn_+)\, h_v$$

ONE local operator (for π)

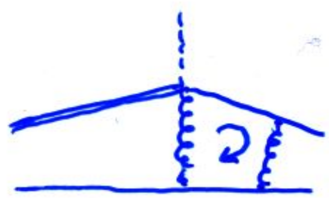
short-distance coefficients

a non-local operator (light-cone)

$\tau \leftrightarrow$ momentum fraction



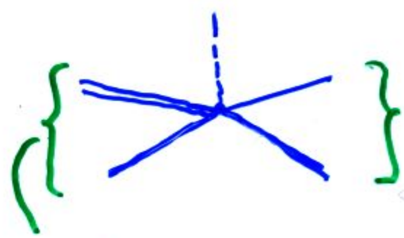
Repeat these steps for the $SCET_I$ operators.
Consider the second term



Integrate out →

short-distance (hard-collinear) fluctuations

Scale $\sqrt{m_b \Lambda}$



Power counting

$$\int \frac{d(2Er)}{2\pi}\, e^{-2iEr\tau}\,\bar\xi A_{\perp}^{(hc)}(rn_+)\, h_v$$

at leading order in $1/m_b$ for $B\to\pi$

$$= \int d\omega\, dv\, J(\tau; v, \omega)\left[\bar\xi(sn_+)\frac{\not n_+}{2}\gamma_5\xi\right]_{FT}\left[\bar q_s(tn_-)\frac{\not n_-}{2}\gamma_5 h_v\right]_{FT}$$

short-distance coefficient

$\downarrow$ $\phi_\pi(v)$ — $\langle \pi | \ldots | 0\rangle$

$\downarrow$ $\phi_B(\omega)$ — $\langle 0 | \ldots | \bar B\rangle$

soft-collinear factorization!

matrix element of $\bar{\xi} h_v$

(MB, Feldmann '00 ; '03 ; Lange, Neubert '03)

$$\underbrace{f_{+,0,T}}_{i}(E) = C_i^{(A0)}(E)\,\xi(E)$$

$$+ \int d\omega\, dv \left[\int d\tau\, C_i^{(B1)}(E,\tau)\, J(\tau; v, \omega) \right] \phi_{B(\omega)} \phi_{\pi(v)}$$

tree (Charles et al. '98)
1-loop (MB, Feldmann '00
Bauer et al '00)

tree $\simeq \mathcal{O}(\alpha_s)$ (MB, Feldmann '00)
1-loop (MB, Kiyo, Yang '04,
MB, Yang '05;
Hill et al. '04
Kirilin '05)

$\xi(E)$ unknown

$$f_+ \longrightarrow \xi \longrightarrow f_0, f_T$$

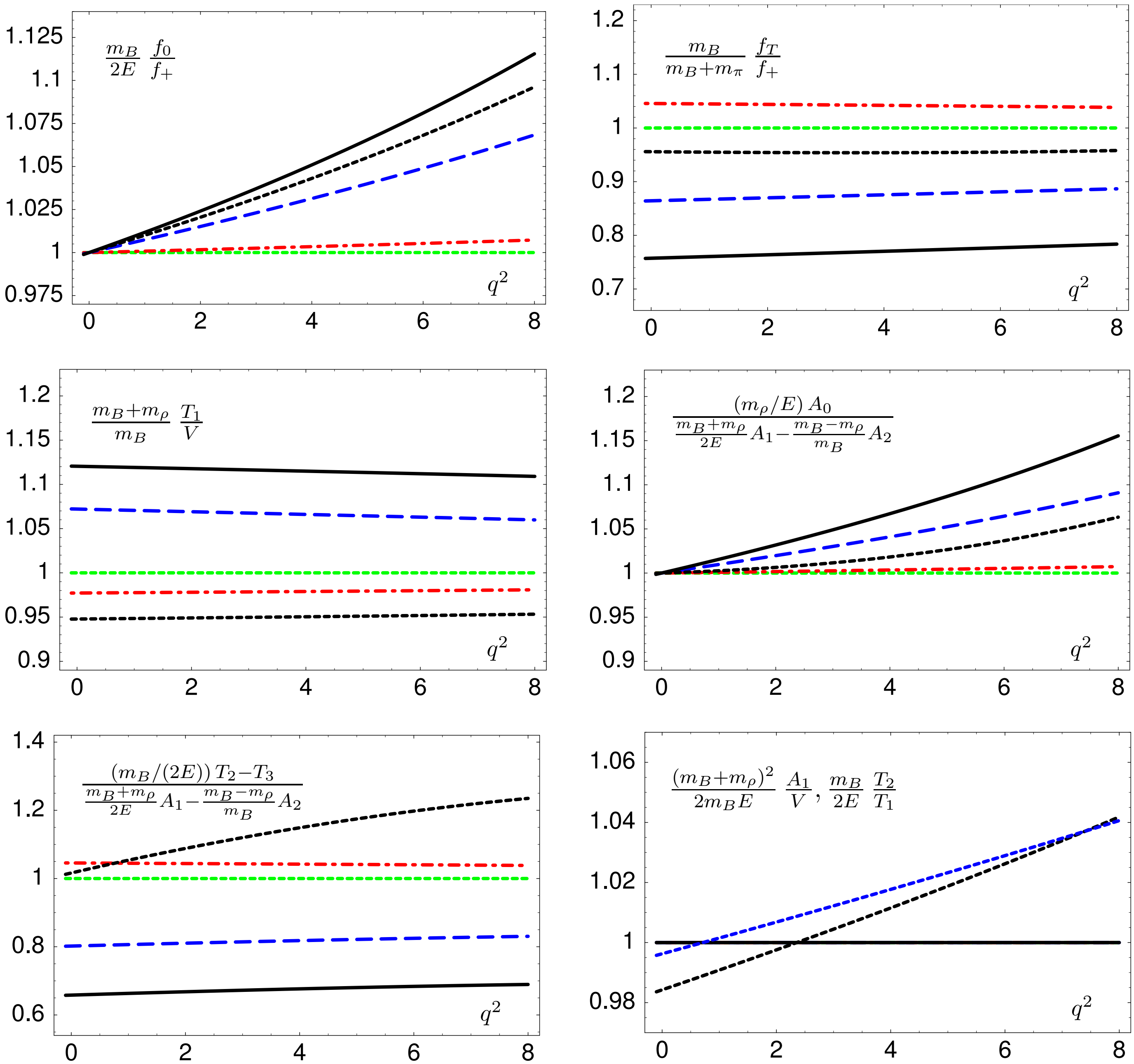
QCD SR
data?

vector mesons

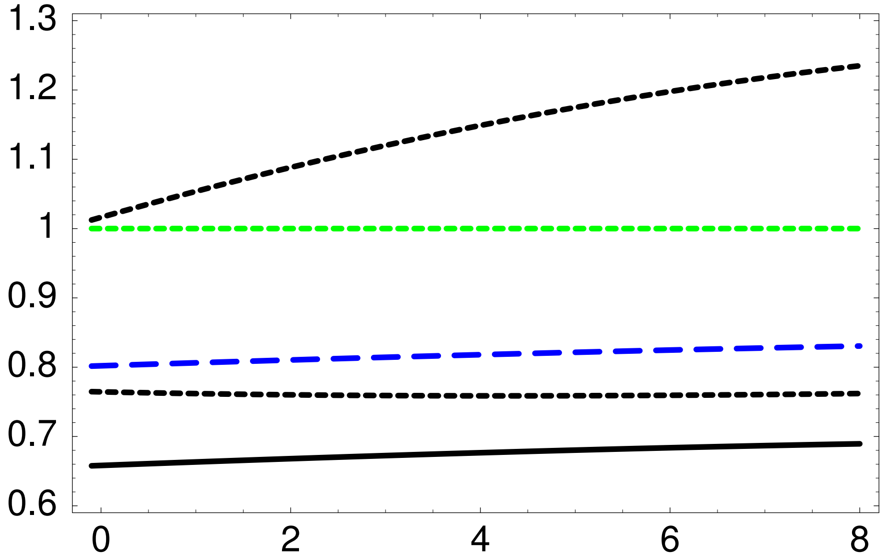
$$\underset{\text{"}A_1 - A_2\text{"}}{V} \longrightarrow \xi_{\parallel}, \xi_{\perp} \longrightarrow A_0, \text{"}A_1 + A_2\text{"}, T_1, T_2, T_3$$

$\int d\omega dv \ldots$ must converge!

- only $\int \frac{d\omega}{\omega} \ln^n \omega\, \phi_B(\omega)$ can appear (power counting)
  finite (proof?)
- if $\int dv$ were endpoint-divergent one would need a regulator to factorize s from c. But there is no soft divergence
- confirmed in 1-loop calculations



Corrections to the $B \to \pi$ and $B \to \rho$ form factor ratios as a function of q^2. The ratios equal 1 in the absence of radiative corrections. Solid (black) line: full result, including NLO and resummed leading-logarithmic correction to spectator-scattering; Long-dashed (blue): without NLO+LL correction to spectator-scattering; Dash-dotted (red): without any spectator-scattering term. Dashed (black): QCD sum rule calculation. The lower right panel shows the two form factor ratios that equal 1 at leading power. For comparison, the QCD sum rule results for these two ratios are shown (upper (blue) line refers to A_1/V, lower (black) line to T_2/T_1.



Ball/Braun98 (lower black-dashed) vs Ball/Zwicky04 (upper black-dashed) for $\frac{m_B}{2E}\,T_2(E) - T_3(E)$ divided by $\frac{m_B+m_V}{2E}\,A_1(E) - \frac{m_B-m_V}{m_B}\,A_2(E)$.

Open problems

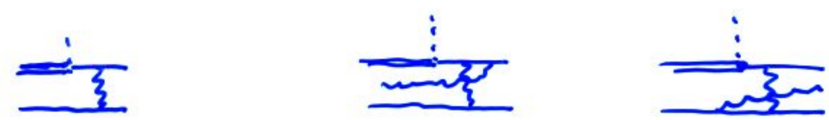
* implications of this for QCD sum rules
  ($1/m_b$ expansion, twist-expansion, what is $\phi'_\pi(1)$, light-cone dominance?)

  $\longrightarrow$ Thorsten's talk

* understanding $\langle \pi | \bar\xi h_v | \bar B \rangle_{SCET_I} \sim \xi$
  (large logs? soft overlap? soft-collinear factorization)

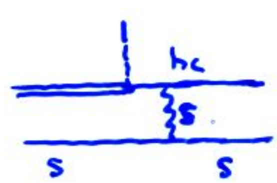
We didn't match $\bar\xi h_v$. Why?

$$\bar\xi h_v \longrightarrow [\bar\xi\xi][\bar q_s h_v] + [\bar\xi A_\perp \xi][\bar q h_v] + [\bar\xi\xi][\bar q A_\perp h_v]$$

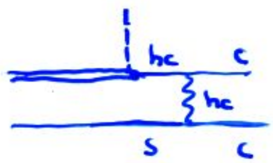


- all leading power
- no twist expansion (3particle amps)
- convolution integrals diverge
  $\hookrightarrow$ soft-collinear factorization breaks down

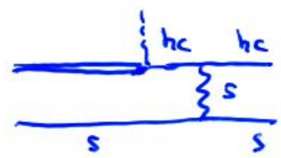
$$\int \frac{du}{u^2}\,\phi_\pi(u)$$ dominated by $u \simeq 0$



Naive picture

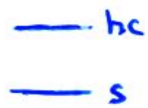


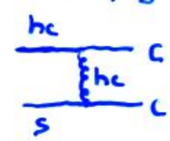
hard-scattering



soft overlap

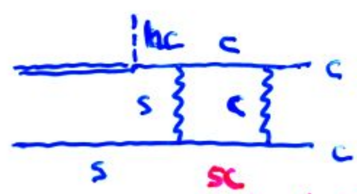
both of the same order

- $\xi(E)$ is not only the soft overlap
- 

does not exist as a configuration in the SCET pion


- or (Becher, Neubert '03)

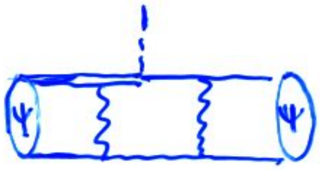
depends on regularization



sc ~ $(\Lambda, \Lambda^{3/2}, \Lambda^2/M)$

Toy model $\quad B_c \to \eta_c \qquad m_c \sim$ soft scale

Muonium → Positronium



$$\propto \alpha_s(\mu_{hc}) \cdot \log\frac{m_b}{m_c} \cdot \log\frac{m_b}{m_c}$$

endpoint logarithm ; hc log

(Bell, Feldmann)

Open problem :

$$\alpha_s(\mu_{hc})^n \log\frac{m_b}{\Lambda} \sim 1$$

$\log\frac{m_\mu}{m_e}$

How to sum these logs ?
Even for $B_c \to \eta_c$ or $[\mu e] \to [ee]$!
Key to understanding factorization & soft overlap !

cf : KLN theorem (1964)

from understanding $\log\frac{m_\mu}{m_e}$ in muon decay