q2e, exp and more

Matthias Steinhauser (University of Hamburg)
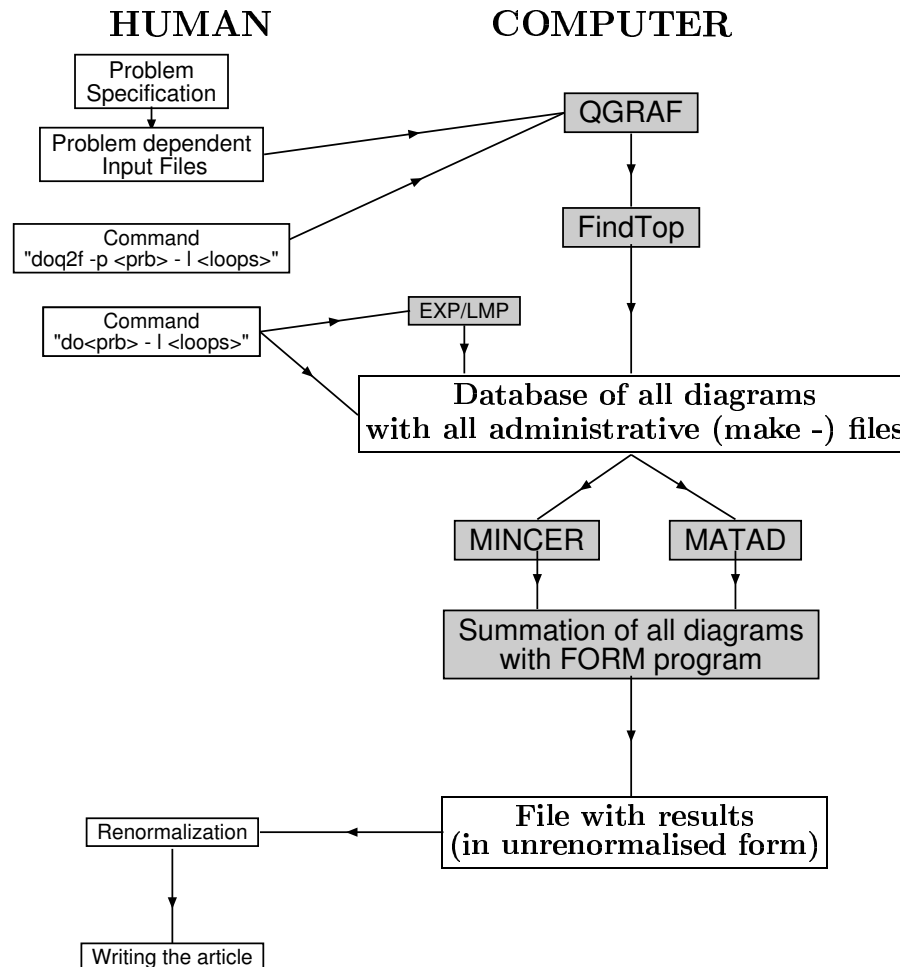
CAMW, Karlsruhe, May 2004

Important features

- Automatic generation of diagrams
- Automatic computation
- Automated asymptotic expansion
- Automated interface between the individual parts
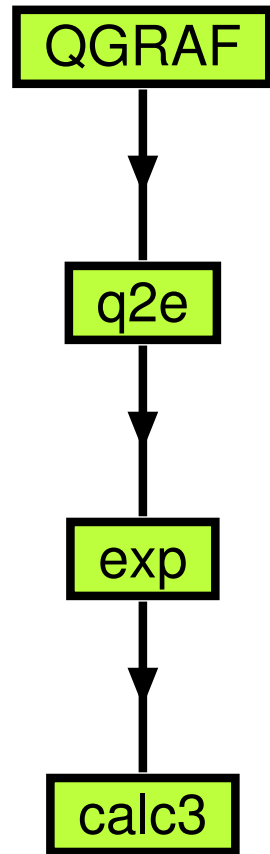- Job control
- Speed

Realization

GEFICOM

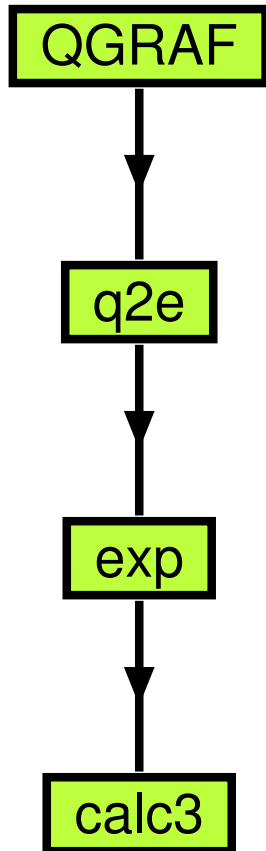automatic GEneration, FInding topologies and COMputation of

Feynman diagrams

| HUMAN | COMPUTER |

- GEFICOM [Chetyrkin,MS'96-'02]
- AWK, PERL, Mathematica, ...

Problem Specification

Problem dependent Input Files

QGRAF

Command "doq2f -p <prb> - l <loops>"

FindTop

Command "do<prb> - l <loops>"

EXP/LMP

Database of all diagrams
with all administrative (make -) files

MINCER

MATAD

Summation of all diagrams
with FORM program

File with results
(in unrenormalised form)

Renormalization

Writing the article

Realization

QGRAF

↓

q2e

↓

exp

↓

calc3

Realization

QGRAF

[Nogueira'93]

q2e

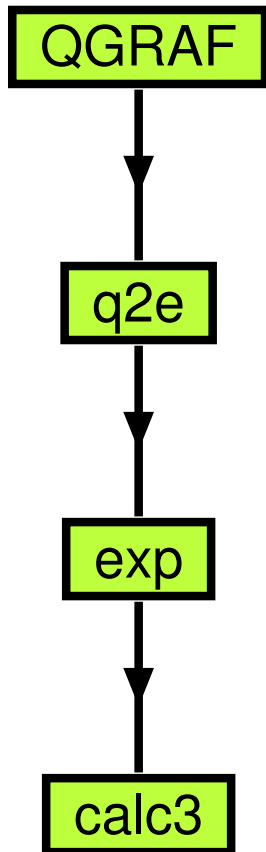[Seidensticker'01]

[Harlander,Seidensticker,MS'98]

exp

[Seidensticker'98-'02]

calc3

➪ default:

MATAD [MS'96-'00]

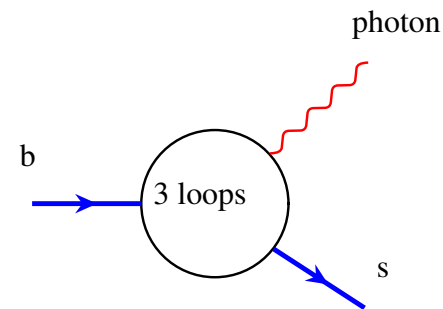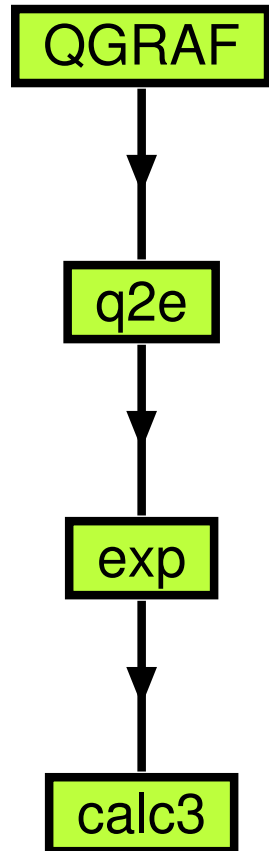MINCER [Larin,Tkachov,Vermaseren'91]

Generation of diagrams

QGRAF ` $+$ fast, flexible

$-$ no pictures, "nothing pre-prepared"

• f77, "black box"

Input: `qgraf.dat`

q2e

exp

calc3

```
output = 'qlist.' ;
style = 'q2e.sty' ;
model = 'c7.lag' ;
in =  fbq,a ;
out = fsq ;
loops = 3 ;
loop_momentum = k ;
options = onepi ;
true = iprop[fcq,0,0] ;
true = iprop[a,0,0] ;
true = iprop[Wp,pp,1,2] ;
```

photon

b

3 loops

s

Generation of diagrams (2)

```
QGRAF
  |
  v
 q2e
  |
  v
 exp
  |
  v
calc3
```

`

Input: <prb>.lag

```
[fcq,fCq,-]
[ftq,fTq,-]
[fsq,fSq,-]
[fbq,fBq,-]
[Wm,Wp,+]
[pm,pp,+]
[a,a,+t]
[g,g,+t]
[c,C,-]
[fCq,fbq,Wp]
[fTq,fbq,Wp]
[fSq,fcq,Wm]
[fSq,ftq,Wm]
[fCq,fbq,pp]
[fTq,fbq,pp]
[fSq,fcq,pm]
[fSq,ftq,pm]
[Wp,Wm,a]
[pp,pm,a]
[fTq,ftq,a]
[fCq,fcq,a]
[fSq,fsq,a]
[fBq,fbq,a]
[C,c,g]
[g,g,g]
[fTq,ftq,g]
[fCq,fcq,g]
[fSq,fsq,g]
[fBq,fbq,g]
```

Output: qlist.

```
{
diagram              593
pre_factor           (+1)*1

number_propagators 9
number_loops        3
number_legs_in      2
number_legs_out     1


external_leg         q1|1|fbq
external_leg         q2|2|a
external_leg         q3|3|fSq


momentum             p1|1,2|fBq,fbq
momentum             p2|3,1|g,g
momentum             p3|2,4|fBq,fbq
momentum             p4|5,3|fSq,fsq
momentum             p5|4,6|fTq,ftq
momentum             p6|4,6|Wp,Wm
momentum             p7|7,5|fSq,fsq
momentum             p8|7,5|g,g
momentum             p9|6,7|fSq,fsq
}
```
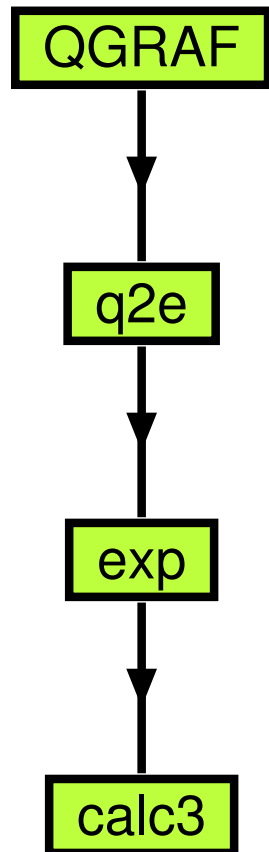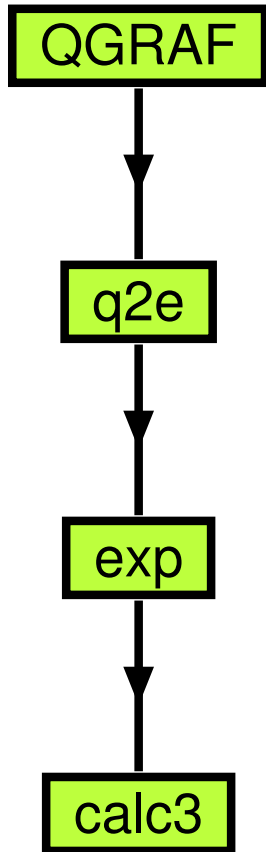
Scales, Feynman rules, ...

QGRAF	• C++
↓	
	Input: <prb>.conf
q2e `	* q2e.propagator_file bsg.prop
↓	* q2e.vertex_file bsg.vrtx
	* q2e.scales M1,M2
	* q2e.mass Wp:M2
	* q2e.mass pp:M2
exp	* q2e.mass ftq:M1
↓	* q2e.mass fbq:M3
	* q2e.expand_naive q1,q2,M3
	* q2e.closed_fermion_loop fsq:nls
	* q2e.closed_fermion_loop fcq:nlc
calc3	* q2e.closed_fermion_loop fbq:nlb
	* q2e.closed_fermion_loop ftq:nht

q2e

QGRAF

q2e

exp

calc3

Input: \<bsg\>.prop
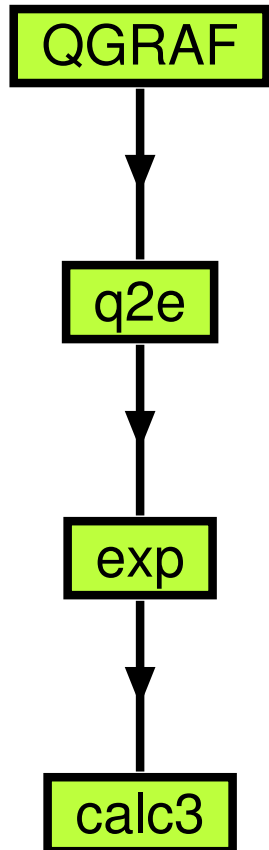
```
{ftq,fTq:*FT<spin_line>(<momentum>)
     |*d_(<spinor_index_vertex_1>,<spinor_index_vertex_2>)||}
{fcq,fCq:*FT<spin_line>(<momentum>)
     |*d_(<spinor_index_vertex_1>,<spinor_index_vertex_2>)||}
{fbq,fBq:*FT<spin_line>(<momentum>)
     |*d_(<spinor_index_vertex_1>,<spinor_index_vertex_2>)||}
{fsq,fSq:*FT<spin_line>(<momentum>)
     |*d_(<spinor_index_vertex_1>,<spinor_index_vertex_2>)||}

{g,g:*Dg(<lorentz_index_vertex_1>,<lorentz_index_vertex_2>,<momentum>)
     |*prop(<colour_index_vertex_1>,<colour_index_vertex_2>)||}
{c,C:*Den(<momentum>)
     |*prop(<colour_index_vertex_1>,<colour_index_vertex_2>)||}

{a,a:*Dg(<lorentz_index_vertex_1>,<lorentz_index_vertex_2>,<momentum>)
     |||}
{Wp,Wm:*DW(<lorentz_index_vertex_1>,<lorentz_index_vertex_2>,<momentum>)
     |||}
{pp,pm:*DPhi(<momentum>)
     |||}
```

---

q2e (2)

QGRAF → q2e → exp → calc3

Input: `<bsg>.vrtx`

```
{fTq,ftq,g:*FT<spin_line>(<lorentz_index_particle_3>)
        |*GM(<colour_index_particle_3>,
            <spinor_index_particle_1>,<spinor_index_particle_2>)||}

{C,c,g:*Vgh(<lorentz_index_particle_3>,<outgoing_momentum_1>)
      |*V3g(<colour_index_particle_1>,<colour_index_particle_3>,
            <colour_index_particle_2>)||}

{fTq,ftq,a:*FT<spin_line>(<lorentz_index_particle_3>)
        |*d_(<spinor_index_particle_1>,<spinor_index_particle_2>)||*Qt}

{g,g,g:*V3g(<lorentz_index_particle_1>,<outgoing_momentum_1>,
            <lorentz_index_particle_2>,<outgoing_momentum_2>,
            <lorentz_index_particle_3>,<outgoing_momentum_3>)
      |*V3g(<colour_index_particle_1>,<colour_index_particle_2>,
            <colour_index_particle_3>)||}

{fTq,fbq,Wp:*FT<spin_line>(<lorentz_index_particle_3>,g7)
        |*d_(<spinor_index_particle_1>,<spinor_index_particle_2>)||*gWptb}
{fSq,ftq,Wm:*FT<spin_line>(<lorentz_index_particle_3>,g7)
        |*d_(<spinor_index_particle_1>,<spinor_index_particle_2>)||*gWmst}

{a,Wp,Wm:*VBaWW(<lorentz_index_particle_1>,<momentum_1>,
               <lorentz_index_particle_2>,<momentum_2>,
               <lorentz_index_particle_3>,<momentum_3>)
      |||*gBaWW}
```
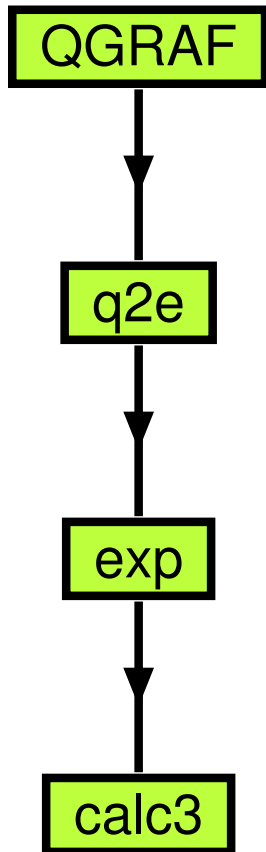
q2e (3)

QGRAF

q2e

exp

calc3

`

Output: `<prb>.dia`

```
*--#[ d3l593 :

        (+1)*1
        *FT1(nu9)
        *FT1(p4)
        *FT1(nu22)
        *FT1(p7)
        *FT1(nu21)
        *FT1(p9)
        *FT1(nu18,g7)
        *FT1(p5)
        *FT1(nu17,g7)
        *FT1(p3)
        *FT1(mu2)
        *FT1(p1)
        *FT1(nu10)
        *Dg(nu9,nu10,p2)
        *DW(nu17,nu18,p6)
        *Dg(nu21,nu22,p8)
        ;

        #define TOPOLOGY "arb"
        #define INT1 "arb"

*--#] d3l593 :
```
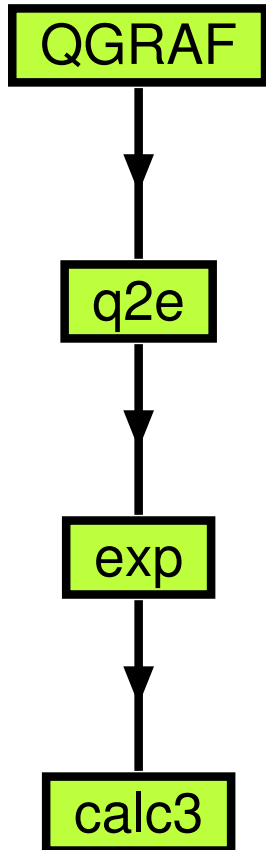
Output: `<prb>.edia`

```
{d3l593;9;3;2;3;M1,M2;(q1,e:1,3)(q2,e:2,3)(p1,M3,e:1,2)(p2:3,1)

(p3,M3,e:2,4)(p4:5,3)(p5,M1:4,6)(p6,M2:4,6)(p7:7,5)(p8:7,5)(p9:6,7)}
```

```
*--#[ fqcd3l593 :

        1
        *GM(b(9),i3,j14)
        *d_(j14,j13)
        *GM(b(22),j13,j20)
        *d_(j20,j19)
        *GM(b(21),j19,j24)
        *d_(j24,j23)
        *d_(j23,j16)
        *d_(j16,j15)
        *d_(j15,j12)
        *d_(j12,j11)
        *d_(j11,j8)
        *d_(j8,j7)
        *GM(b(10),j7,i1)
        *prop(b(9),b(10))
        *prop(b(21),b(22))
        ;

*--#] fqcd3l593 :

*--#[ fqed3l593 :
        1
*--#] fqed3l593 :

*--#[ few3l593 :
        1*gWmst*gWptb*Qb
*--#] few3l593 :
```

M. Steinhauser, University of Hamburg

Asymptotic expansion

QGRAF

q2e

exp

calc3

- Many scale-hierarchies in real life:

  $R(s):$ $m_q \ll \sqrt{s}$

  $Z \to b\bar{b}:$ $m_t \gg M_Z$

  $b \to s\gamma:$ $m_t \gg M_W \gg m_b$

  $\ldots$

- Example:
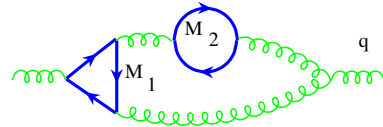
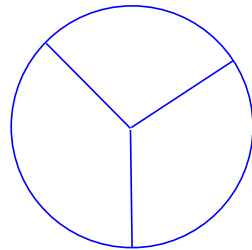 $q^2 \gg M_1^2 \gg M_2^2 \Rightarrow 21$ sub-diagrams

- `lmp`: large-momentum procedure; `PERL` [Harlander'97]

- `exp`: iterated hard-mass- and large-momentum

  procedure; `C++` [Harlander,Seidensticker,MS'98] [Seidensticker'98-'02]

  no limitation of #loops and #legs

  generation of make-files, summation files, ...

exp

QGRAF

q2e

exp
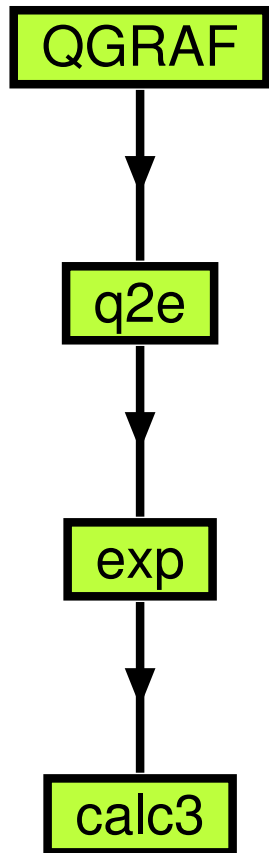
calc3

Input: `topsel.calc3`

```
{top3l,poco_scale,copy_scale,dala;6;3;0;1; ;
    (p1:3,1)(p2:4,3)(p3:3,2)(p4:1,4)(p5:4,2)(p6:2,1);
    111111;011111;101111;110111;111011;111101;111110;001111;010111;011011;
    011101;011110;100111;101011;101101;101110;110011;110101;110110;111001;
    111010;111100;000111;001011;001101;001110;010011;010101;010110;011001;
    011010;011100;100011;100101;100110;101001;101010;101100;110001;110010;
    110100;111000;000011;000101;000110;001001;001010;001100;010001;010010;
    010100;011000;100001;100010;100100;101000;110000;000001;000010;000100;
    001000;010000;100000;000000}
```

Input: `<prb>.GLOBAL`

- Options for `exp` (paths, order of integrations, ...)

- Limits on expansion parameters

- "Special" vertices ("standard" included in `calc3`)

- Projectors

...

exp (2)

QGRAF

q2e

exp

`

calc3
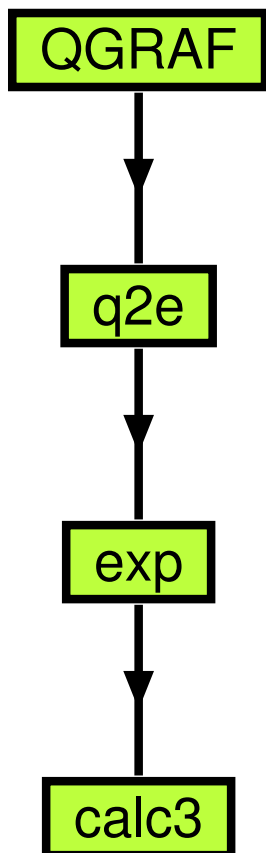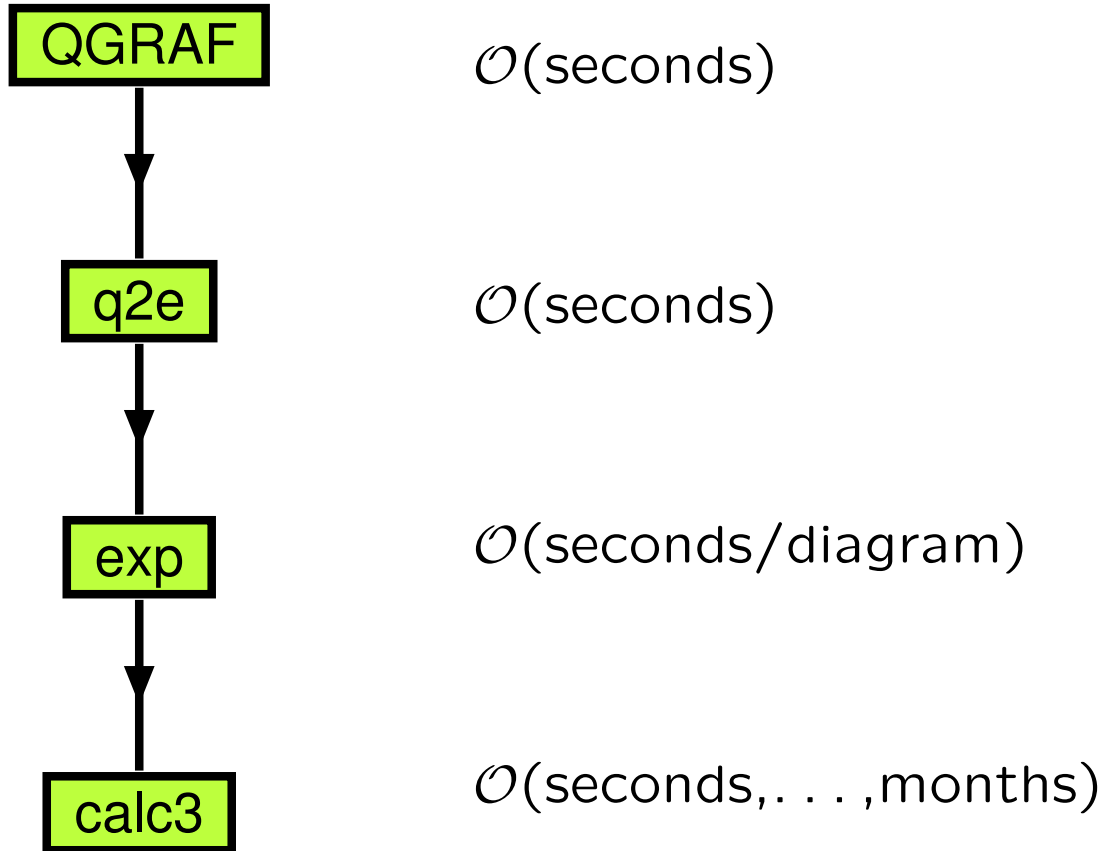
```
*--#[ d3l493_2 :

        (+1)*1
        *FT1(nu14,g7)
        *FT1(L,28,M1,pM1,exp,+p21,pM2,28,L)
        *FT1(nu24)
        *FT1(L,37,+p31,pM1,M1,pM1,37,L)
        *FT1(nu23)
        *FT1(L,34,M1,pM1,exp,+p21,pM2,34,L)
        *FT1(nu11)
        *FT1(L,31,-p12,pM1,M1,pM1,exp,-q1,pQ1,+p21,pM2,31,L)
        *FT1(nu13,g7)
        *FT1(L,19,+p11,pM1,exp,+q2,pQ2,M3,pM3,19,L)
        *FT1(mu2)
        *FT1(L,16,+p11,pM1,exp,M3,pM3,16,L)
        *FT1(nu12)
        *Dg(nu11,nu12,L,22,+p11,pM1,exp,-q1,pQ1,22,L)
        *DW(nu13,nu14,L,25,-p21,pM2,M2,pM2,exp,+q1,pQ1,+q2,pQ2,25,L)
        *Dg(nu23,nu24,L,40,+p32,pM1,exp,+p21,pM2,40,L)
        ;

        #define INT1 "topL1"
        #define MASS1 "M1"
        #define DALA1 "0"
        #define INT2 "topL1"
        #define MASS2 "M2"
        #define DALA2 "0"
        #define INT3 "topL1"
        #define MASS3 "M1"
        #define DALA3 "0"

    *--#] d3l493_2 :
```
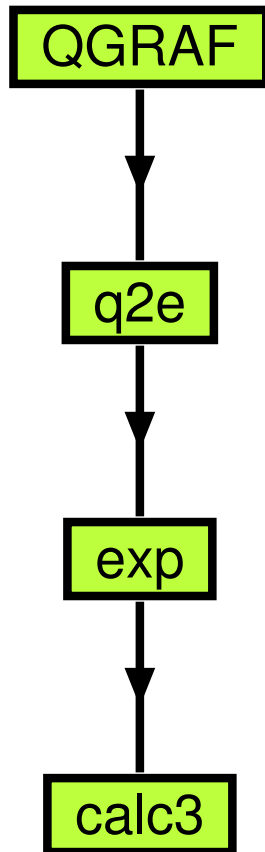
Computation

```
QGRAF
  │
  ▼
 q2e
  │
  ▼
 exp
  │
  ▼
calc3
```
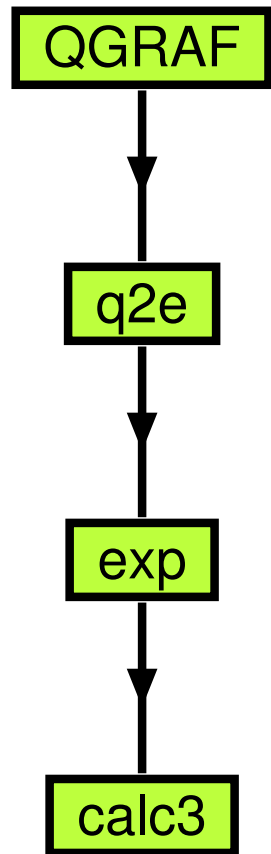
- `FORM` [Vermaseren]

- Interpretation of $<$`prb`$>.<$`#`$>.$`src` files:
  explicit expressions for vertices, propagators,
  traces, expansions, . . .

- "Real integration" :

MATAD: vacuum ints. with 1 mass scale up to 3 loops

MINCER: massless 2-point functions up to 3 loops,

ON-SHELL2 [Kalmykov'99]: on-shell ints. up to 2 loops,

. . .   [whatever you can compute]

- Laporta, . . .

- Interface to `Mathematica`

Typical runtime

QGRAF	$\mathcal{O}(\text{seconds})$
q2e	$\mathcal{O}(\text{seconds})$
exp	$\mathcal{O}(\text{seconds/diagram})$
calc3	$\mathcal{O}(\text{seconds},\ldots,\text{months})$

Input/output

QGRAF	Input:	`qgraf.dat`
		`<mod>.lag`
	Output:	`qlist.`

q2e	Input:	`<mod>.prop`
		`<mod>.vrtx`
		`<prb>.conf`
	Output:	`<prb>.dia`
		`<prb>.edia`

exp	Input:	`topsel.calc3`
		`<prb>.GLOBAL`
	Output:	`<prb>.<#>.src`

calc3	Output:	sum of bare diagrams

Successful applications

- $H \to gg$, 3 loops, QCD, [Chetyrkin, Kniehl,MS'97]
- $e^+e^- \to$ hadrons, $\mathcal{O}(\alpha_s^2)$, $\sqrt{s} \gg m_q$, [Chetyrkin,Harlander,Kühn,MS,97]
- $Z \to b\bar{b}$, $\mathcal{O}(\alpha\alpha_s)$, [Harlander,Seidensticker,MS,98]
- $\overline{\text{MS}}$–on-shell relation, 3 loops, QCD, [Chetyrkin,MS,99]
- 2-loop static potential, [Kniehl,Penin,Smirnov,MS'02]
- ρ parameter, 3 loops, $\mathcal{O}((G_F m_t^2)^3)$ [Faisst,Kühn,Seidensticker,Veretin'03]
- $b \to s\gamma$, 3 loop matching, [Misiak,MS'04]

. . .